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Original Article

Statistical sampling method to verify the homogeneity of full-scale cement-solidified radioactive waste

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ABSTRACT

Homogeneity is an important factor for ensuring the structural stability of solidified radioactive waste, and the most effective approach for assessing its homogeneity is by performing compressive strength measurements using the minimum amount of coring specimens. The efficiency of detecting inhomogeneous waste is affected by the coring position and number of coring positions. However, no guidelines exist for coring solidified waste for compressive-strength tests. Therefore, this study compared uniform, random, and quasi-Monte Carlo sampling methods to determine the most effective core position. Further, the effects of different sampling amounts on the detection rate of inhomogeneous solidified waste were observed, and the detection rate of the inhomogeneous waste was obtained by modeling the coring procedure of solidified radioactive waste using MATLAB. Thus, a sampling method and a method for increasing the specimen amount, both of which can efficiently detect inhomogeneous waste during compressive strength tests, were presented in this paper. The results of this study can be applied as background data for developing homogeneity assessment guidelines for solidified radioactive waste.

1. Introduction

Most radioactive waste repositories require fluid waste to be solidified per the waste acceptance criteria (WAC) [1–4], which considers structural stability (represented by compressive strength) in an expected disposal environment as a critical requirement. When developing the solidification process, well-established standards such as ASTM C39 are employed for laboratory-scale specimens of solidified waste (e.g., Φ5 × 10 cm) smaller than the actual waste disposed of at the repository (e.g., 200-L size drum) to investigate if they conform to the WAC [2,5–7]. Laboratory-scale specimen experiments can adequately verify the compatibility between the solidifying material (e.g., cement and polymer) and waste.

The waste should be homogeneous in the drum, and its material characteristics should correlate with those obtained in the lab-scale experiments [8–10] because the waste and solidifying material may not mix uniformly in the drum, causing significant variations in the material characteristics such as strength degradation [11]. Thus, these requirements must be satisfied to dispose of full-size solidified waste. Cement-solidified waste is a representative example that is widely applied for radioactive waste and exhibits large variations among specimens [5,12–18]. The homogeneity and other characteristics of full-scale waste are evaluated using specimens obtained by coring and sectioning; however, no standard sampling procedures have been established to obtain representative specimens [19–21].

Although several sampling methods have been developed in other fields, they cannot be applied because of the relatively small size of the waste drum (Φ61.5 × 88.4 cm for 200-l drum) compared to that of the cored specimen (Φ5 × 10 cm). The most common sampling methods include random (RND) and uniform (UNI) sampling [22–26]; however, they may not be effective for a small number of samples. A quasi-random sampling method, such as Hammersley's algorithm, is a more statistically strategic approach for determining the coring positions. Kalagnanam and Diwekar [27] revealed that a sampling technique based on Hammersley points can reduce the computational intensity of stochastic optimization problems. Further, Lee et al. [28] demonstrated that the Hammersley sequence requires a lower number of points to reach a certain level of accuracy compared to that when using random and uniform samples. Although the quasi-random sampling method appears effective, its statistical effectiveness needs to be examined before being adapted for solidified radioactive waste.

Given this background, this study aims to identify the most effective

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sampling method to detect the inhomogeneity of radioactive waste. A three-dimensional (3D) radioactive waste model was created using MATLAB to compare the efficiencies of the uniform, random, and Hammersley sampling methods for detecting inhomogeneous solidified waste. From the modeling, the detection rates of inhomogeneous solidified waste according to the sampling methods were obtained. Further, a method for increasing the amount of specimen required to obtain the highest increase in detection rate was derived. This study presents the most effective sampling method and a method for increasing the specimen amount to efficiently detect the inhomogeneity of radioactive waste. The results of this study can be applied as background data for developing homogeneity assessment guidelines for solidified radioactive waste.

2. Methods

2.1. Hammersley sampling

Hammersley sampling is a quasi-Monte Carlo method based on the Hammersley sequence, which is a low-discrepancy sequence [28–30]. The positive integer q can be expressed using the prime base p [30] as

$$q = a_0 + a_1 p + a_2 p^2 + \ldots + a_r p^r \ (a_i \in [0, p-1]). \tag{1}$$

The function Φp (radical inverse) for Eq. (1) can be defined as

$$\Phi_p(q) = \frac{a_0}{p} + \frac{a_1}{p^2} + \ldots + \frac{a_r}{p^{r+1}}. \tag{2}$$

Any sequence for p represents the sequence for Φp. The corresponding Hammersley point with the q-th d-dimension is given as

$$\left(\frac{q}{N}, \Phi_{p_1}(q), \Phi_{p_2}(q), \ldots, \Phi_{p_{d-1}}(q) \right) (q = 0, 1, 2, \ldots, N-1). \tag{3}$$

where $p_1 < p_2 < \ldots p_{d-1}$ and N represents the total number of Hammersley points.

Assuming the two-dimensional (2D) Hammersley point from Eq. (3),

$$x_i = \frac{i}{N} \ (i \in 1, 2, \ldots, N-1),\ y_i = \sum_{j=0}^{k-1} \left(\left\lfloor \frac{i}{p^j} \right\rfloor Mod\, p \right) \times 2^{-(j+1)} \left(k = \left\lceil log_p N \right\rceil \right). \tag{4}$$

The van der Corput sequence can be applied for computing by assuming p = 2 in Eq. (4). Fig. 1 shows ten Hammersley points on a 2D plane.

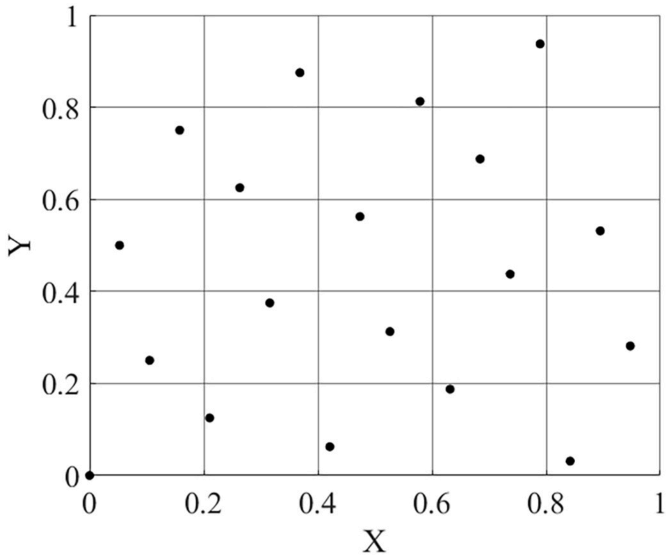

Fig. 1. Ten Hammersley points on a 2D plane.

To apply Eq. (4) for a cylindrical surface, the Euclidean (x,y) coordinates are converted to polar (r, φ) coordinates [28] as

$$r_i = y_i^{\frac{1}{2}} \times R, \phi_i = x_i \times 360^o. \tag{5}$$

where R represents the radius of the circle. The center of the circle is considered the starting point of sampling. Assuming that the edge point of the circle is the starting point of the sampling, Eq. (5) can be converted to

$$r_i = (1 - y_i)^{1/2} \times R, \phi_i = x_i \times 360^o. \tag{6}$$

Ten Hammersley points in the 2D plane determined using Eqs. (5) and (6) are illustrated in Fig. 2.

2.2. Geometry of solidified waste and abnormal region

The geometry of the solidified radioactive waste follows a 200 L drum with a 61.5 cm diameter and height of 88.4 cm, which is the most commonly used geometry in Korean repositories. The geometry of the solidified radioactive waste was generated using 266,490 data points with a 1 cm distance between points, as shown in Fig. 3. As the distance between data points is 1 cm, the height of the solidified waste in the model is set to 88 cm, 0.4 cm shorter than that of actual waste. Further, as shown in Fig. 4, the X-, Y-, and Z-coordinate information is stored as a matrix (position matrix) in the order of data point creation to facilitate data point selection.

The position, size, and number of abnormal regions (i.e., inhomogeneous regions) in the actual waste are expected to be random, and therefore, the position of the abnormal regions is set at random. The size and number of abnormal regions follow specific values of the modeling conditions to control the level of inhomogeneity. As shown in Fig. 3, spherical abnormal regions are generated to describe the abnormal region inside the solidified waste, and the positions of the abnormal regions change every time a drum geometry is created. As shown in Fig. 4, the random positions of the abnormal regions are generated by randomly selecting numbers in a sequence matrix connected to the position matrix.

According to the test condition, the number of abnormal regions is set as 1–3; based on the diameter of the actual waste (D), the diameters of the abnormal regions are set to 6.15 (D/10), 8.2 (D/7.5), and 12.3 cm (D/5). A sliced sphere is generated when an abnormal region is placed near the surface; if the abnormal region overlaps, the sphere is relocated to another position. An indicator that can distinguish abnormal points from data points is required to detect abnormal points inside the waste. Thus, as illustrated in Fig. 4, the position matrix of the data points in the waste geometry is initially connected to a homogeneity matrix with a value of one. The homogeneity matrix connected to the position matrix of the abnormal region has a value of zero, and the final uniformity matrix of the waste reflecting the abnormal region is derived based on the position matrix of the abnormal region (Fig. 4).

2.3. Coring position based on the sampling method

In this study, RND and UNI sampling methods were compared with the Hammersley sampling method, and therefore, modeling was performed for four sampling methods: 1) Hammersley-center (HC), 2) Hammersley-edge (HE), 3) random (RND), and 4) uniform (UNI). Only vertical coring applied to the top surface of the waste was considered, and the diameter of the coring was set to 5 cm, following the WAC of the Korean repository. Thus, based on the position derived from the sampling methods, coring specimens consisting of data points were generated.

The coring positions of the HC and HE sampling methods were derived using Eqs. (5) and (6), respectively. The coring positions for the

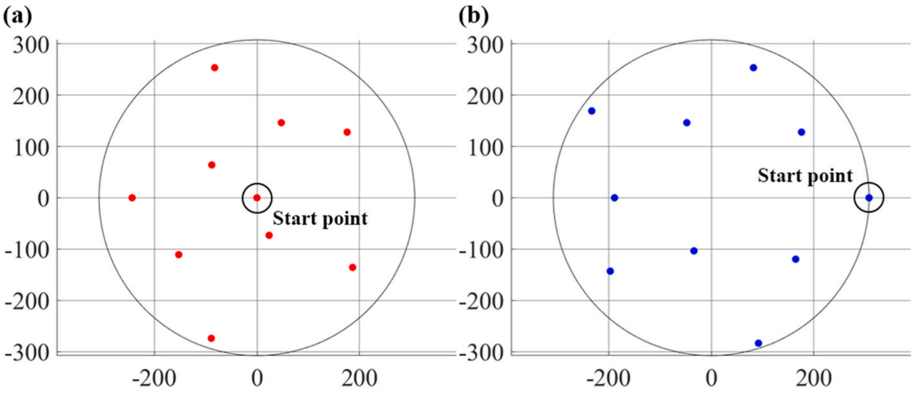

Fig. 2. Ten Hammersley points on the circle where the sampling start points are at (a) the center of the circle and (b) at the edge of the circle.

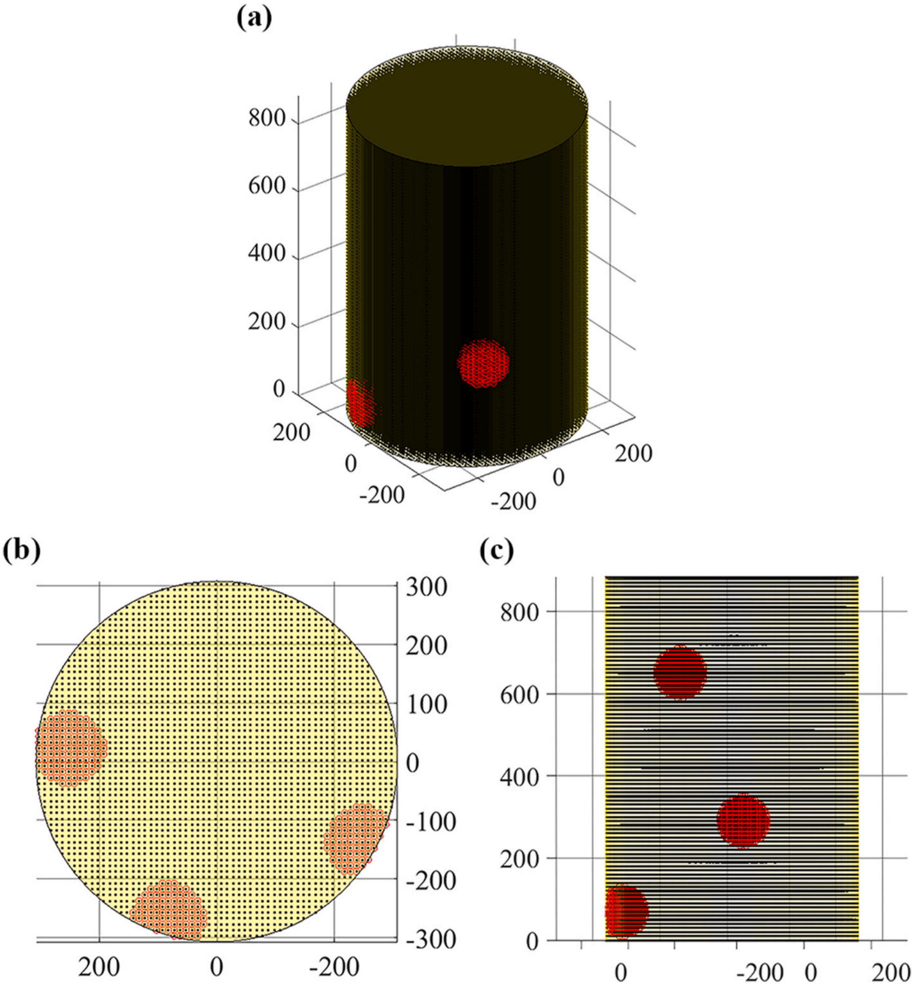

Fig. 3. Geometry waste including abnormal region: a) 3D geometry, b) top view, and c) side view.

RND sampling method were generated by applying time as a seed in a random function to generate different positions for each calculation. In the RND sampling method, the coring position was recalculated using a while-if loop when the coring position overlapped or was placed out of geometry. For the UNI sampling method, the coring positions were uniformly distributed in the circle, including at the center of the circle. Following the number of coring positions used for the UNI sampling method, the number of coring positions in the model was set to 4, 7, 13, and 19. The coring positions for each sampling method based on the number of coring positions are shown in Fig. 5.

2.3.1. Modeling input parameters

The calculation needs to be performed for a sufficient number of samples to compensate for the randomness of the position of the abnormal region inside the waste. A sensitivity study comparing the detection rate of the abnormal region based on the total number of samples was conducted to determine the sufficient number of samples (number of coring positions: 19, number of specimens per core: 5, diameter of the abnormal region: 12.3 cm, and number of abnormal regions: 3). The results of the sensitivity study are shown in Fig. 6. A sufficiently small detection rate change was observed when the number

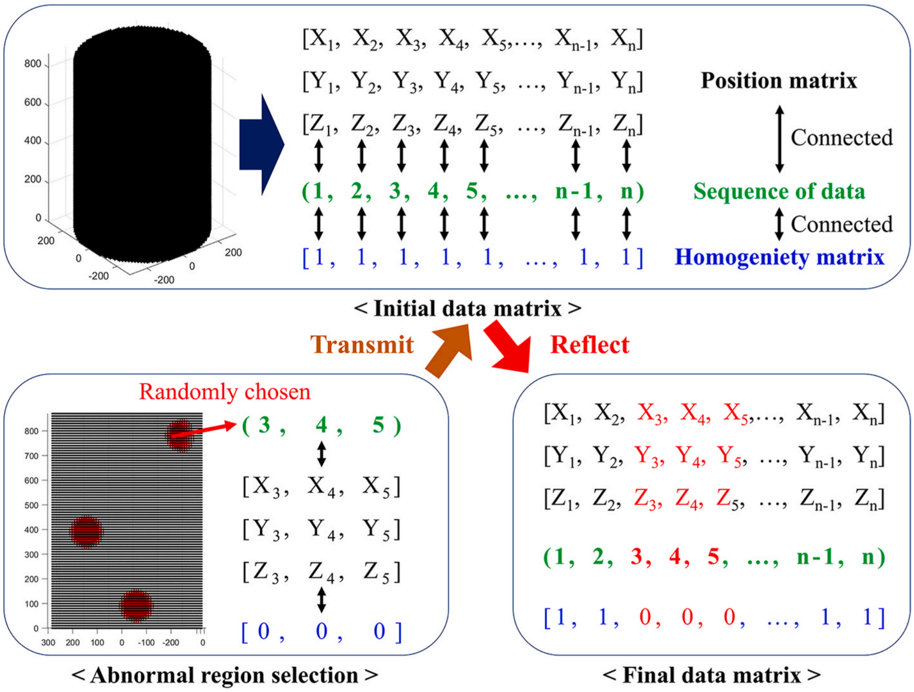

Fig. 4. Composition of data point matrix inside the solidified waste.

of samples was larger than 40,000. Therefore, in this study, the number of samples was set to 100,000 to compensate for the randomness.

As illustrated in Fig. 7, modeling is performed in this study to compare the detection rates for each sampling method and the increase in the detection rate based on the method used to increase the specimen amount. The detection rates of all sampling methods under all modeling conditions were calculated to compare the detection rates based on the sampling method. As shown in Fig. 7, the number of specimens can be increased by increasing the number of coring positions (horizontal direction) or specimens per core (vertical direction).

When the number of specimens per core increases, the vertical position of the specimen according to the number of specimens per core follows

$$(\mathbf{88.4} - \mathbf{10} \times \boldsymbol{N}) \div (\boldsymbol{N} - \mathbf{1})\ \boldsymbol{cm}\ (\boldsymbol{N} : \boldsymbol{Number\ of\ specimen\ per\ core}). \quad (7)$$

The average detection rate of all sampling methods was applied when comparing the increase in the detection rate based on the method used for increasing the number of specimens. The detection rate based on the method used for increasing the number of specimens was derived by increasing the number of cores (or number of specimens per core) and fixing the number of specimens per core (or number of coring positions). The increase in the detection rate per specimen was derived via linear regression analysis, and the average detection rate increase per specimen based on the method used for increasing the specimen amount was obtained.

2.3.2. *Criterion for abnormal region detection*

A criterion for abnormal region detection was established to calculate the detection rate of inhomogeneous solidified waste. The assumption that abnormal regions of solidified waste cannot be visually detected but can only be detected by compressive strength measurements was applied before establishing the criterion. As illustrated in Fig. 8, the data points for the cored specimens were obtained after setting the coring position following the sampling method. In this study, the specimen was assumed to follow the compressive strength of the abnormal region when 10 % of the data points inside the coring specimen showed values from the homogeneity matrix of the abnormal region. Therefore, among the coring specimens derived for each sampling method, the sampling method was used to detect the inhomogeneous solidified waste if at least one specimen with 10 % abnormal region data points was detected.

3. Results

3.1. *Detection rate according to the sampling method*

Modeling was performed for 100,000 solidified wastes for each modeling condition. The results, displayed in Fig. 9 based on the number of coring positions when the minimum number of specimens per core was three specimens. Fig. 9 shows the sampling method that achieves the maximum detection rate based on the modeling conditions; the square brackets and parentheses present the maximum detection rate and difference between the maximum and minimum detection rates, respectively. The detection rate difference between the maximum and minimum values based on the sampling method was in the range of 0.1–8.8 %. A difference larger than 5 % occurred when the number of coring positions was greater than 13. Among the sampling methods, the HC sampling method showed the maximum detection rate in 10 out of 27 cases when the total number of specimens was less than 40 (number of coring positions ≤13). The UNI sampling method showed a maximum detection rate in 7 out of 9 cases in the other cases (number of coring positions = 19).

The sampling method that shows the maximum detection rate when the number of coring positions is at the minimum (i.e., four coring positions), and its value according to the number of specimens per core (vertical direction), are shown in Fig. 10. The difference between the maximum and minimum detection rates based on the sampling method was in the range of 0.1–2.4 %. Further, the HC sampling method showed the maximum detection rate in 15 of 27 cases, and the UNI sampling method showed the maximum detection rate in 6 of 27 cases.

The overall results of the modeling, including the result when the number of coring positions or the number of specimens per core is fixed to the minimum value, are summarized in Table 1. Among the 108 cases, the sampling methods that showed the maximum detection rate were ranked as follows: UNI (56 cases), HC (29 cases), HE (20 cases), and RND (3 cases); when the total number of specimens was less than 40, they

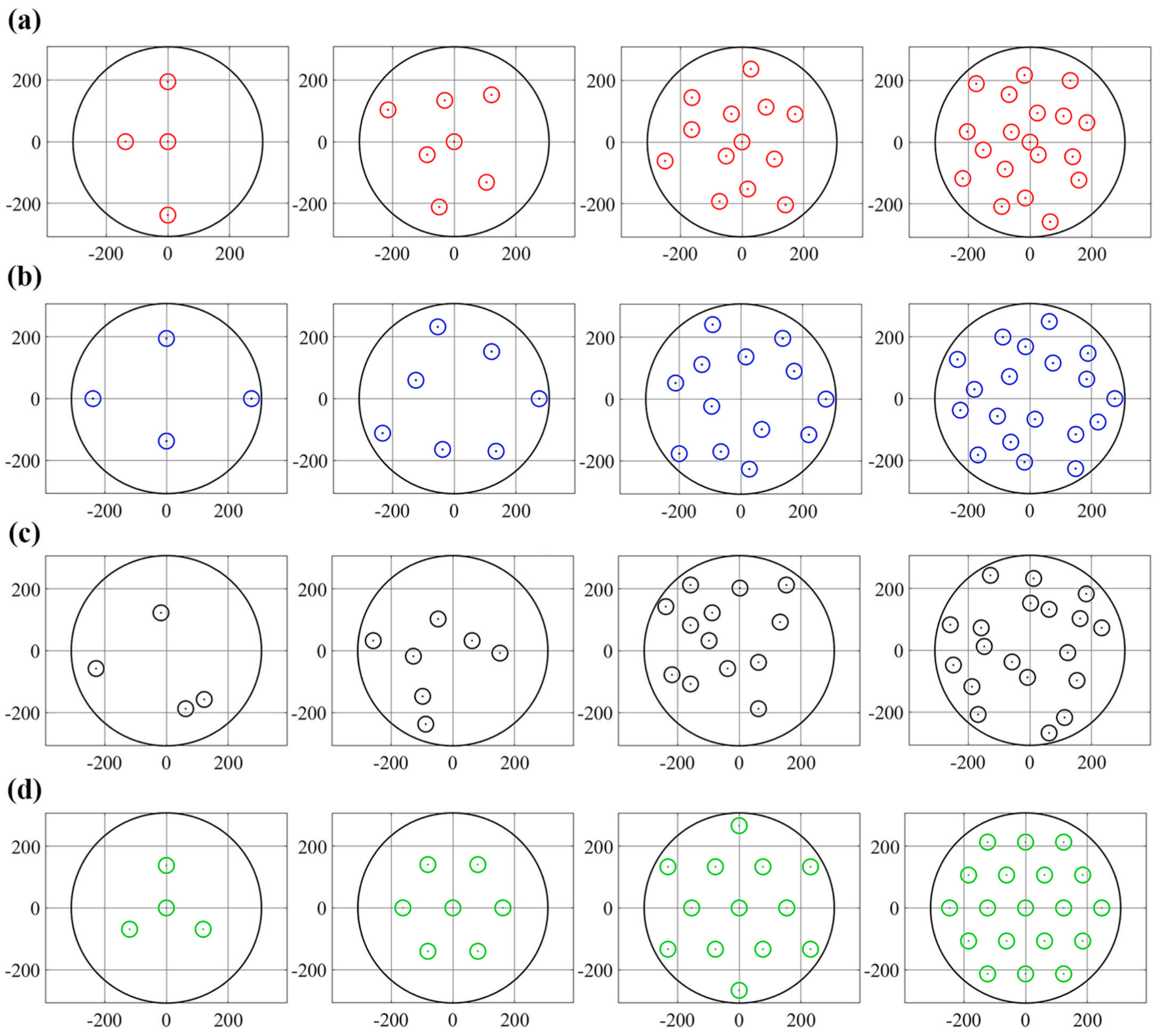

Fig. 5. Coring positions for the (a) Hammersley-center (HC), (b) Hammersley-edge (HE), (c) random (RND), and (d) uniform (UNI) sampling methods when the number of coring positions is 4, 7, 13, and 19.

were ranked as HC (25 cases), UNI (22 cases), HE (13 cases), and RND (3 cases); and when more than 40 specimens were used, they were ranked as UNI (34 cases), HE (7 cases), HC (4 cases), and RND (0 cases).

3.2. Effect of method used for increasing the specimen amount on the detection rate

To determine the effect of the increase in the specimen amount on the detection rate, the average increases in the detection rate were calculated according to the number of coring positions when the number of specimens per core was fixed (total three for each D and number of abnormal regions) and the number of specimens per core when the number of coring positions was fixed (total four for each D and number of abnormal regions). An increase in the detection rate was obtained using the linear regression method; the minimum value of R^2 was larger than 0.9. Fig. 11 shows the increase in the detection rate of the abnormal regions with an increasing number of specimens.

The difference in the increase in detection rate based on the method used to increase the amount of specimen increased with an increase in the number and size of the abnormal regions. The difference in the increase in the detection rate based on the method used for increasing the specimen amount was below 15 % when the size of the abnormal region was smaller than 8.2 cm. For an abnormal region diameter of 12.3 cm, the increase in the detection rate attributed to the increase in the number of specimens per core was ~50 % higher than that when the number of coring positions was increased.

4. Discussion

The detection rate of inhomogeneous solidified waste was derived by modeling solidified waste with abnormal regions (inhomogeneous part of the waste) at random positions using MATLAB. The abnormal region detection results indicate that the HC and UNI sampling methods exhibited the highest performance. The HE sampling method, which follows the same Hammersley theory, showed a lower detection rate than those of the HC and UNI sampling methods, whereas the RND

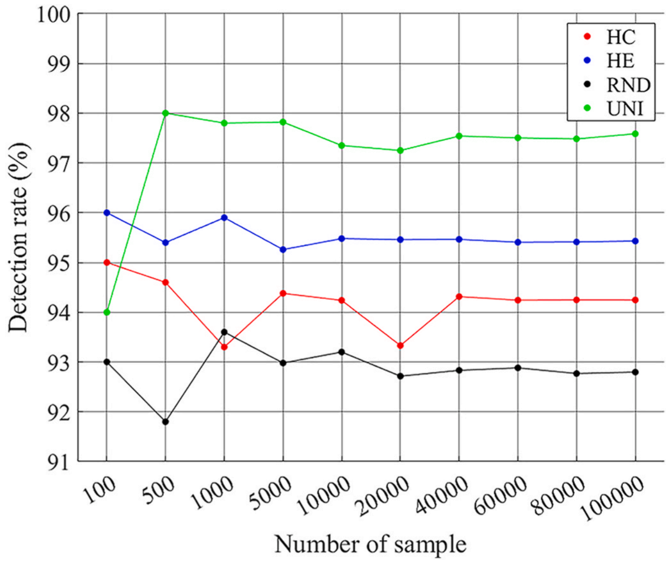

Fig. 6. Sensitivity test results of the abnormal region detection rate based on the sample number.

sampling method showed the lowest detection rate in most cases. The difference between the HC and UNI sampling methods and the HE and RND sampling method is that the former always extract the coring specimen at the center position of the waste, regardless of the number of coring positions. However, the HE sampling method extracts the coring specimen near the center of the waste when the number of coring positions is sufficiently large, and the extraction of the specimen from the center of the waste using the RND sampling method is uncertain because of its randomness. Thus, including the center of the waste as a coring position is essential to effectively detecting the inhomogeneous solidified waste through coring specimens.

Minimizing the number of coring specimens for the test can help confirm if the WAC are satisfied. Based on this preference, the detection rate of abnormal regions is determined by fixing the number of coring positions or specimens per core. The uniform sampling method showed the highest detection rate in most cases in the calculations based on the number of coring positions when the number of specimens per core was fixed at the minimum value (three). As shown in Fig. 9, the sampling method with the highest detection rate varied in most cases according to the total number of specimens. The UNI sampling method showed the highest detection rate in most cases when the number of specimens was >40, whereas the HC sampling method showed the highest detection rate in most cases when the number of specimens was <40. In the calculation based on the number of cores, the HC sampling method showed the highest detection rate in most cases for waste with a small size and a low number of abnormal regions (total number of specimens less than 40) when the number of specimens per core was fixed to a minimum (Fig. 10). Thus, HC sampling appears to be the most efficient method to minimize the number of specimens when the number of coring positions or number of specimens per core is the minimum; however, the difference in the detection rates between the HC and other sampling methods is small. Table 1 indicates that the UNI sampling method showed the highest detection rate in 56 of 108 cases; however, the HC sampling method was found to be the most effective method when the number of specimens was less than 40. The UNI sampling was found to be the most effective method when the number of specimens was greater than 40. Thus, regardless of the number of coring positions or specimens per core, the HC sampling method is the most effective method for detecting inhomogeneous waste when the number of specimens is small. Further, the UNI sampling method became more effective than the HC sampling method when the number and size of the abnormal regions increased. This tendency indicates that the former is more effective with an increase in the inhomogeneity of the waste. Thus, the UNI sampling method is the most effective when there is no limit on the number of specimens and the inhomogeneity of the waste is large, whereas the HC sampling method is slightly more effective than other methods when the inhomogeneity of the waste is low.

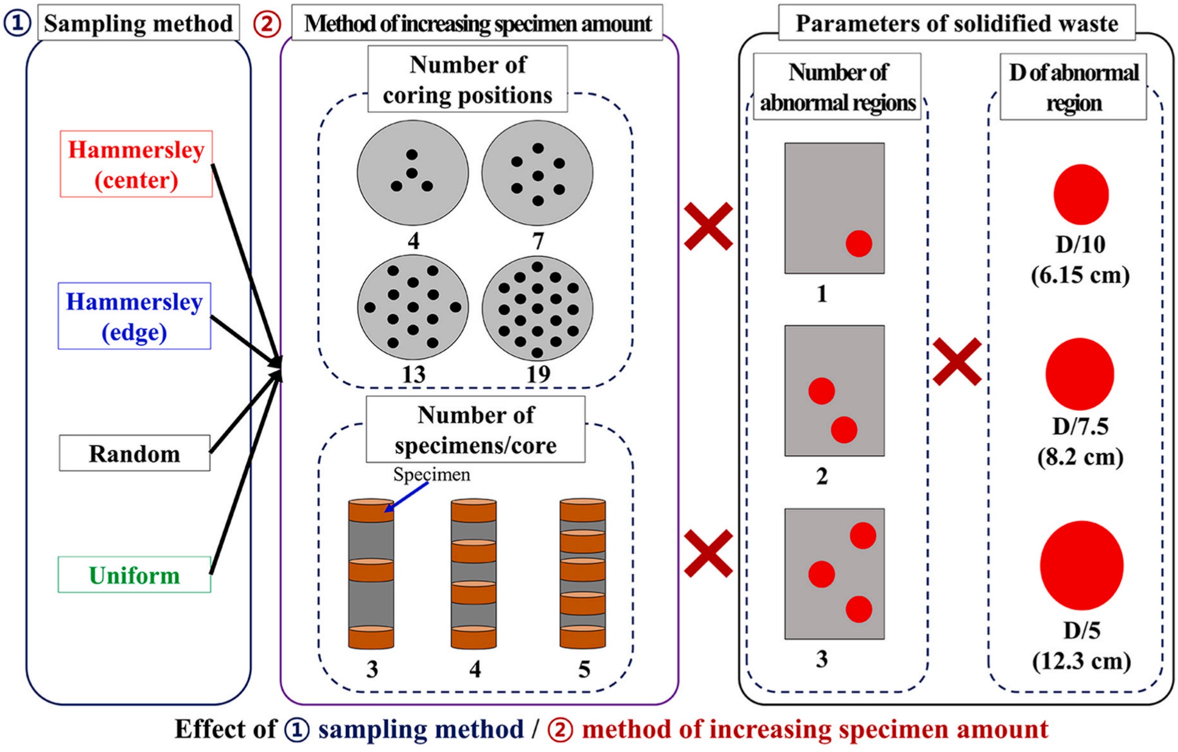

Fig. 7. Research flow for comparing the effects of the sampling method and increase in the specimen number on the detection rate.

The number of coring specimens is limited, and the size and number

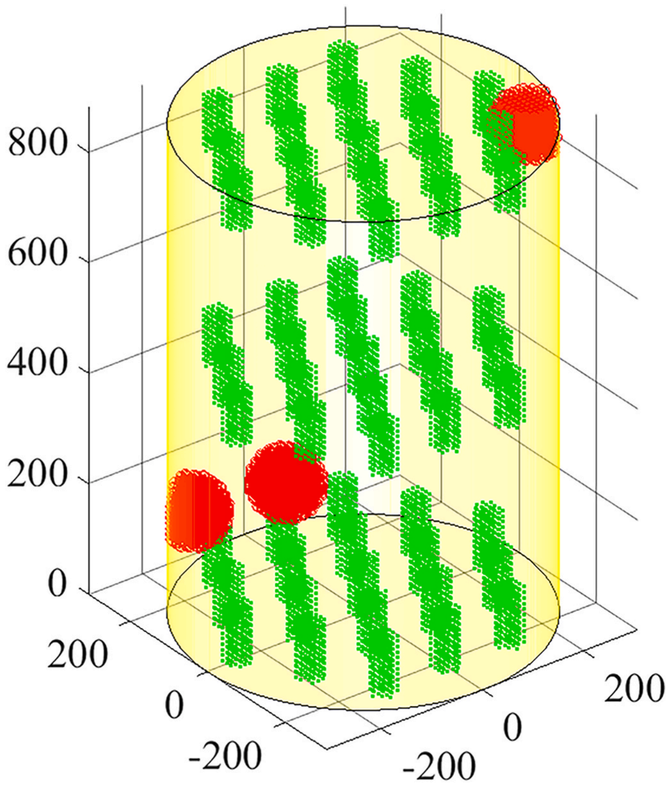

Fig. 8. Example of data points for coring specimens (sampling method: UNI, number of coring positions: 13, number of specimens per core: 3).

of abnormal regions are completely random in an actual coring procedure for cement-solidified radioactive waste. Therefore, applying the HC sampling method for coring seems adequate when considering solidified waste with low inhomogeneity. On the other side, the difference in the detection rate between the HC and UNI sampling methods for waste with low inhomogeneity was sufficiently small. In cases where the HC sampling method showed the highest detection rate, the maximum detection rate difference between the HC and UNI sampling methods was 0.35 %, and its maximum relative difference (= detection rate difference between maximum and minimum/maximum detection rate) was 3.1 %. However, when the UNI method showed the highest detection rate, the maximum detection rate difference between UNI and HC sampling was 9.3 % and the maximum relative difference was 13.5 %. The difference in the detection rates of the HC and UNI sampling methods indicates that the difference between the HC and UNI sampling methods is sufficiently small to neglect waste with low inhomogeneity, thereby implying that the UNI sampling method is significantly effective for wastes with high inhomogeneity. The results mean that UNI sampling method can effectively detect the inhomogeneous waste with both low and high inhomogeneity while HC sampling method is less efficient for the waste with high inhomogeneity. Therefore, by additionally considering the simplicity of the geometry of the UNI sampling method compared to that of the HC sampling method, the former which can efficiently detect the inhomogeneous waste with both low and high inhomogeneity was considered the most effective method for coring solidified waste.

Fig. 11 shows the effect of increasing the amount of specimen on the detection rate. When the diameter of the abnormal region was 6.15 cm, the effect of increasing the specimen amount was less than 0.05 %, which was negligible. When the diameter of the abnormal region was 8.2 cm, the effect was <0.1 %. However, for an abnormal region diameter of 12.3 cm, the differences in the increase in the detection rate according to the method used for increasing the specimen amount were 0.187, 0.353, and 0.508 for one, two, and three abnormal regions, respectively. The results indicate that the effect of increasing the number of specimens is negligible for wastes with low inhomogeneity; the method has a significant effect on wastes with high inhomogeneity. Therefore, when deciding the number of total specimens for compressive strength measurements, increasing the number of specimens per core while fixing the number of coring positions is an effective approach for detecting inhomogeneous solidified waste.

The most effective sampling method and the method used for increasing the number of specimens were derived by modeling using MATLAB. The performances of the sampling methods were compared based on the modeling results, and the results indicated that UNI sampling method was the most effective for assessing the WAC of waste. A comparison of the methods used for increasing the number of specimens revealed that increasing the number of specimens per core while fixing the number of cores was more effective for detecting inhomogeneous waste. The minimum number of cored specimens was preferred during the coring and compressive strength measurement procedures. Further, cement-solidified radioactive waste is expected to have large randomness of abnormal regions, which can be severe than the solidified waste assumed in the present study. Thus, following the UNI sampling method for coring solidified radioactive waste is considered the most desirable approach to select the coring position and adjust the number of total specimens based on the number of specimens per core. Based on the results of this study, the measurement of compressive strength and the assessment of homogeneity are expected to be efficiently performed simultaneously for solidified radioactive waste.

In this study, inhomogeneous solidified waste was modeled using MATLAB, the position of the abnormal region was randomly selected, and the size and number of abnormal regions followed the modeling conditions. However, the size and number of abnormal regions in the actual solidified radioactive waste were expected to be larger than those under the modeling conditions. Therefore, the assumed inhomogeneity of the solidified waste can be too conservative compared to that of actual inhomogeneous solidified waste, and thus, inhomogeneous solidified waste data should be collected and reflected in future modeling to avoid over- or underestimation. Further, the specimen was determined to be inhomogeneous when 10 vol% of the specimen contained an abnormal region in the model. However, a value of 10 % was assumed in this study because of the non-existence of the inhomogeneity criterion, and the possibility of visually detecting abnormal regions was excluded. Thus, the detection of inhomogeneous solidified waste performed in this study included conservative assumptions that the abnormal region cannot be visually detected and an inhomogeneity criterion value that is too low may be applied. In future work, additional modeling should be performed to reflect the adequate criterion of inhomogeneity and scenarios where an abnormal region is visually detectable.

5. Conclusions

Modeling was performed using MATLAB to compare the detection rates of the inhomogeneous solidified radioactive waste. Among the sampling methods used in the study, the HC sampling method showed the best performance for waste with a low level of inhomogeneity. However, the detection rate difference between the HC and other sampling methods was sufficiently small to be neglected, whereas the UNI sampling method showed the best performance for the waste, which was clearly inhomogeneous with a significant difference in detection rate compared to that of the other methods. The results mean that the UNI sampling method is efficient for detection of inhomogeneous waste with both low and high inhomogeneity, unlike HC sampling method. The effect of increasing the sample amount on the detection rate was also examined, and the results indicated that increasing the number of samples per core is more effective than increasing the number of cores when the level of inhomogeneity increases. Thus, applying the UNI sampling method to the coring position was assessed to be adequate for effectively detecting the inhomogeneous solidified waste during the

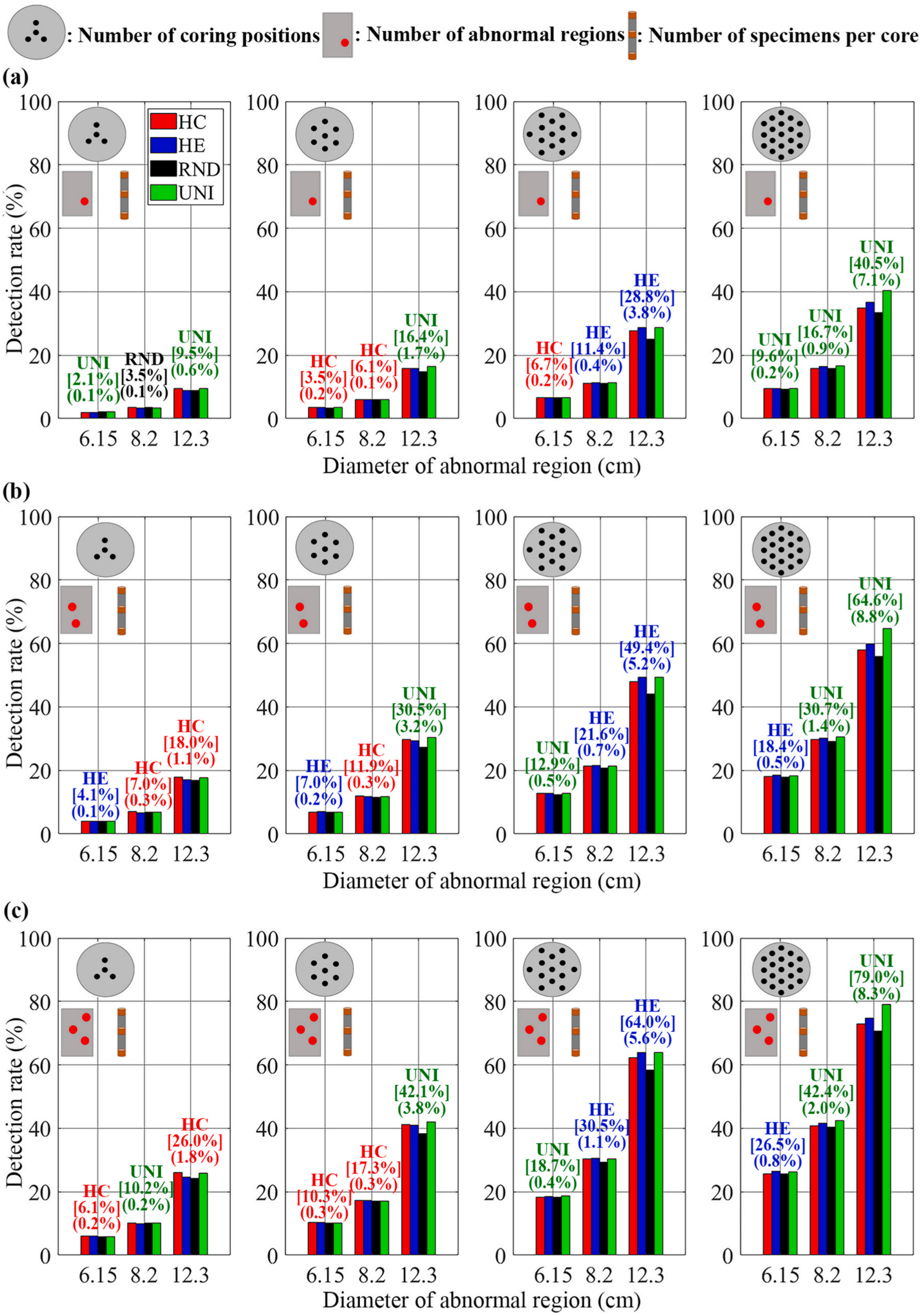

Fig. 9. Detection rate based on the sampling method, and the sampling method with the maximum detection rate when the number of specimens per core is minimum (three) and the number of the abnormal region is (a) one, (b) two, and (c) three (square bracket: maximum detection rate, parentheses: difference between the maximum and minimum detection rates).

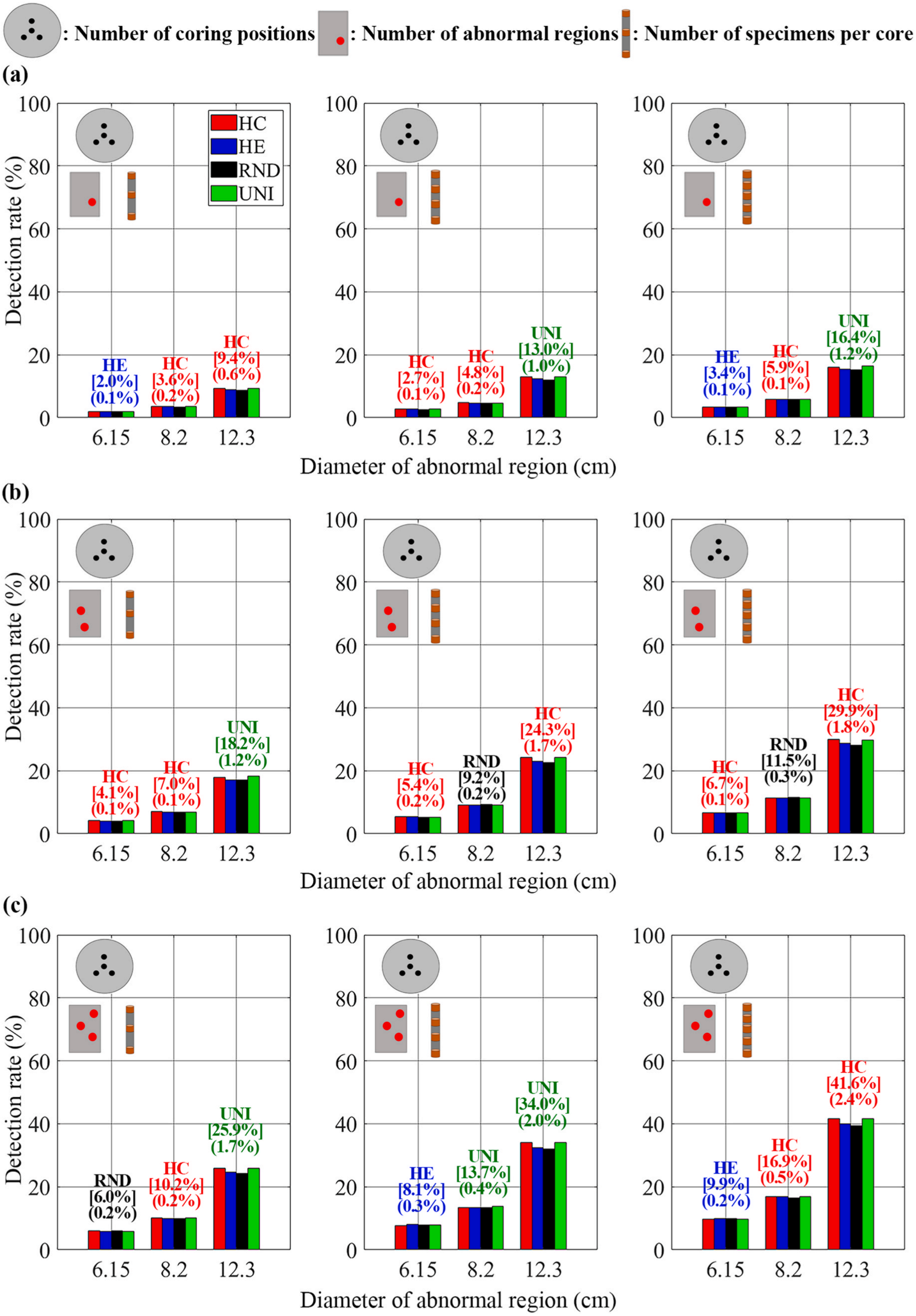

Fig. 10. Detection rate based on the sampling method, and the sampling method with the maximum detection rate when the number of coring positions is the minimum (four) and the number of abnormal regions is (a) one, (b) two, and (c) three (square bracket: maximum detection rate, parentheses: difference between the maximum and minimum detection rates).

Table 1
Sampling methods showing the maximum detection rate according to the modeling conditions.

Number of abnormal regions		1			2			3		
Diameter of abnormal region (cm)		6.15	8.2	12.3	6.15	8.2	12.3	6.15	8.2	12.3
V[b] **= 3** [Max] (%) (Max–Min) (%)	H[a] = 4	**UNI** [2.1][c] (0.09)[d]	**RND** [3.5] (0.1)	**UNI** [9.5] (0.6)	**HE** [4.1] (0.1)	**HC** [7.0] (0.3)	**HC** [18.0] (1.1)	**HC** [6.1] (0.2)	**UNI** [10.2] (0.2)	**HC** [26.0] (1.8)
	H = 7	**HC** [3.5] (0.2)	**HC** [6.1] (0.1)	**UNI** [16.4] (1.7)	**HE** [7.0] (0.2)	**HC** [11.9] (0.3)	**UNI** [30.5] (3.2)	**HC** [10.3] (0.3)	**HC** [17.3] (0.3)	**UNI** [42.1] (3.8)
	H = 13	**HC** [6.7] (0.2)	**HE** [11.4] (0.4)	**HE** [28.8] (3.8)	**UNI** [12.9] (0.5)	**HE** [21.6] (0.7)	**HE** [49.4] (5.2)	**UNI** [18.7] (0.4)	**HE** [30.5] (1.1)	**HE** [63.9] (5.6)
	H = 19	**UNI** [9.6] (0.2)	**UNI** [16.7] (0.9)	**UNI** [40.4] (7.1)	**HE** [18.4] (0.5)	**UNI** [30.7] (1.4)	**UNI** [64.6] (8.8)	**HE** [26.5] (0.8)	**UNI** [42.4] (2.0)	**UNI** [79.0] (8.3)
V = 4 [Max] (%) (Max–Min) (%)	H = 4	**HC** [2.7] (0.1)	**HC** [4.8] (0.2)	**UNI** [13.0] (1.0)	**HC** [5.4] (0.2)	**RND** [9.2] (0.2)	**HC** [24.3] (1.7)	**HE** [8.1] (0.3)	**UNI** [13.7] (0.4)	**UNI** [34.0] (2.0)
	H = 7	**HE** [4.8] (0.1)	**UNI** [8.4] (0.3)	**UNI** [22.7] (2.3)	**HE** [9.4] (0.4)	**HC** [15.9] (0.4)	**UNI** [40.5] (4.1)	**UNI** [13.7] (0.3)	**HC** [22.9] (0.5)	**UNI** [54.0] (6.0)
	H = 13	**HC** [8.9] (0.2)	**UNI** [15.4] (0.3)	**UNI** [39.5] (5.3)	**UNI** [16.9] (0.1)	**HC** [28.5] (0.5)	**UNI** [63.2] (6.4)	**HC** [24.4] (0.2)	**HC** [39.6] (1.1)	**UNI** [77.8] (6.0)
	H = 19	**UNI** [12.9] (0.1)	**UNI** [22.8] (1.3)	**UNI** [56.2] (10.1)	**HE** [24.2] (0.4)	**UNI** [40.3] (1.9)	**UNI** [80.6] (9.8)	**HE** [34.1] (0.4)	**UNI** [53.8] (2.1)	**UNI** [91.3] (7.1)
V = 5 [Max] (%) (Max–Min) (%)	H = 4	**HE** [3.4] (0.1)	**HC** [5.9] (0.1)	**UNI** [16.4] (1.2)	**HC** [6.7] (0.1)	**RND** [11.5] (0.3)	**HC** [29.9] (1.8)	**HE** (9.9) (0.2)	**HC** [16.9] (0.5)	**HC** [41.6] (2.4)
	H = 7	**UNI** [6.0] (0.1)	**HC** [10.5] (0.3)	**UNI** [28.8] (3.4)	**HC** [11.6] (0.2)	**HC** [19.8] (0.4)	**UNI** [49.3] (4.7)	**HC** [17.0] (0.2)	**UNI** [28.7] (0.7)	**UNI** [64.2] (4.8)
	H = 13	**HE** [11.1] (0.1)	**UNI** [19.4] (0.7)	**UNI** [50.0] (6.4)	**UNI** [21.0] (0.3)	**HE** [35.1] (0.6)	**UNI** [74.8] (6.5)	**UNI** [29.8] (0.2)	**UNI** [47.6] (0.9)	**UNI** [87.5] (5.5)
	H = 19	**HE** [16.4] (0.3)	**UNI** [28.4] (1.5)	**UNI** [70.5] (12.0)	**UNI** [29.8] (0.3)	**UNI** [48.9] (2.3)	**UNI** [91.4] (9.0)	**UNI** [41.3] (0.5)	**UNI** [63.4] (2.5)	**UNI** [97.6] (4.7)

[a] Number of coring positions (H).
[b] Number of specimens per core (V).
[c] Maximum detection rate.
[d] Detection rate difference between the maximum and minimum rates.

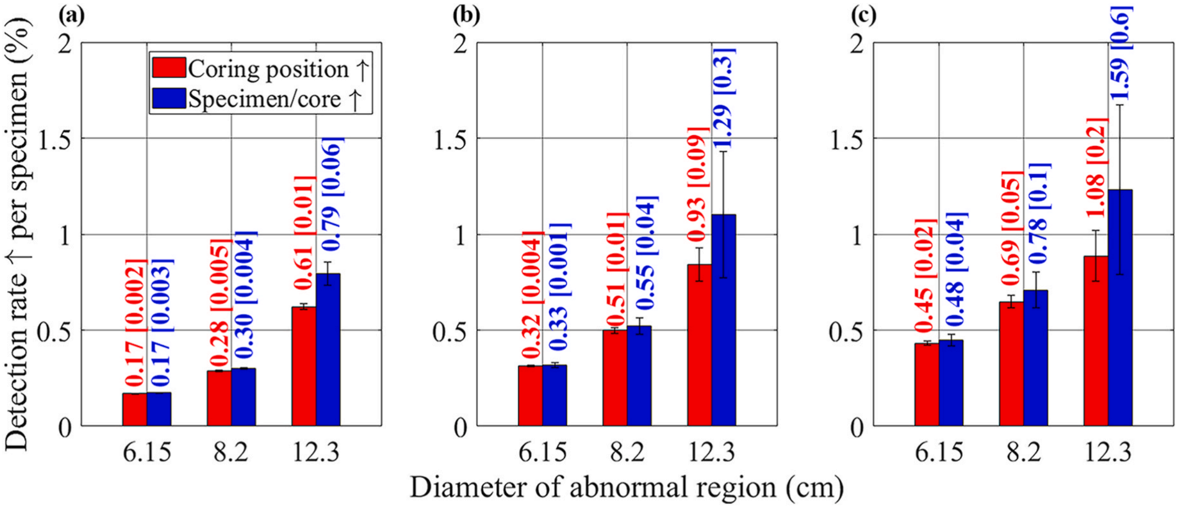

Fig. 11. Increase in the detection rate of abnormal regions based on the method of increasing the number of specimens when the number of abnormal regions is a) one, b) two, and c) three (Detection rate: Average of all sampling methods, square bracket: standard deviation).

compressive strength test of solidified radioactive waste. In addition, considering the number of specimens per core was found to be the most effective method to determine or increase the total number of coring specimens. The results of this study confirm that an efficient waste acceptance criteria procedure can be established through the efficient detection of inhomogeneous solidified waste during the compressive strength test.

CRediT authorship contribution statement

Hyeongjin Byeon: Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Ugyu Jeong:** Writing – review & editing, Software, Methodology, Investigation. **Jaeyeong Park:** Writing – review & editing, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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