Master's Thesis

System Reliability-based Design Optimization under Fatigue-induced Sequential Failure Using Genetic Algorithm

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Department of Urban and Environmental Engineering
(Disaster Management Engineering)

Graduate School of UNIST

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ABSTRACT

Various structures are exposed to the risk of fatigue-induced failure. When a structural system does not satisfy the requirement of structural redundancy, a local failure induced by fatigue may initiate sequential failure, which can cause structural collapse. For an optimized design of such a structural system, it is necessary to quantify the risk of fatigue-induced sequential failure at a system-level and analyze the critical failure sequences. However, it is a challenging task because it is often expensive. This paper proposes a new method for the system reliability-based design optimization (SRBDO) of structures. To consider fatigue-induced sequential failure during SRBDO, the proposed method introduces the Branch-and-Bound method employing system reliability Bounds, termed the B^3 method. By employing the branch-and-bound method, which has been widely adopted for analyzing the failure sequences of a structural system, the B^3 method provides the upper and lower bounds of the system failure probability and identifies the critical failure sequences effectively; this enables the SRBDO of a structure at a low computational cost. Furthermore, in this research, the SRBDO analysis is performed using the genetic algorithm, which is one of the most widely-used optimization methods. The genetic algorithm can determine an optimized solution efficiently by repeatedly creating generations of design variables based on algorithms of selection, crossover, and mutation. In the proposed method, the genetic algorithm is combined with the B^3 method to improve the efficiency of the SRBDO of a structural system subjected to fatigue-induced sequential failure. The proposed framework is demonstrated by applying it to a numerical example of a multi-layer Daniels system, and the corresponding analysis results are presented and discussed.

Keywords: genetic algorithm, system reliability, branch-and-bound, design optimization

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1. Introduction

Numerous methods of structural reliability have been developed to consider the uncertainties underlying structural problems in various engineering fields (Moan 2005, Frangopol and Maute 2003, Thoft-Christensen 1998, Haldar 2006). Particularly, various analytical methods have been developed to estimate the probability of a failure event. However, in numerous cases, the failure of a structure needs to be described by "system reliability" problems. For example, if it is necessary to consider the impact of load re-distribution during a sequential failure, the related system failure event needs to be expressed by a complex function of several component-level limit-states and the related variables.

The risk assessment of fatigue-induced failure requires complicated system reliability analysis (Lee and Song, 2011, 2012). For various structures, fatigue is known to be one of the most critical factors causing structural failure. To analyze fatigue-induced failure and calculate its probability, numerous researchers have developed several reliability analysis methods. However, a number of the available methods focus on the failure probabilities of individual structural members. Such component-level failure probabilities are not sufficient to take into consideration structural redundancy, which can trigger sequential failures and thereby cause the failure of the structural system. Therefore, to obtain an accurate failure probability of the structural system, it should be computed at a system-level.

However, it is not a straightforward task because it is often expensive. The branch-and-bound method has been widely employed to identify the critical fatigue-induced sequential failures and to quantify the risk of fatigue-induced sequential failure. Inspired by the branch-and-bound method, the branch-and-bound method employing system reliability bounds (termed the B^3 method) was recently developed. The B^3 method is one of the branch-and-bound methods, which enables one to carry out the upper and lower bounds of the system failure probability of a structure and to obtain the critical failure sequences accurately and efficiently (Lee & Song 2011, 2012, Lee 2012).

Such a structural system should be designed and maintained to possess an adequate level of structural redundancy to prevent the initial local fatigue-induced failures from causing exceedingly large damages (e.g., system collapse) that are likely to result in catastrophic socio-economic losses. In addition, it is preferred to design a structure optimally. Considering these aspects, the reliability-based design optimization (RBDO) of a structure is often aimed at minimizing the structural material cost while ensuring the targeted level of structural performance. Researchers have studied reliability-based design optimizations for structural safety analysis.

According to Lin et al. (2011), RBDO problems have been investigated for several decades. A number of previous studies focused on the concept of Hasofer and Lind (Hasofer and Lind 1974), who first developed the first order reliability method (FORM) and introduced the reliability index (which is the shortest distance from the origin to the target failure domain in the uncorrelated standard normal space) as a measure of failure probability. Subsequently, the reliability problem can also be considered as an optimization problem, and numerous RBDO methods utilizing the reliability index have been introduced as the reliability index approach (RIA) (Grandhi and Wang 1998, Carter 1997, Frangopol and Corotis 1996, Wu and Wang 1996, Chandu and Grandhi 1995, Enevoldsen and Sorensen 1994, Enevoldsen 1994, Nikolaidis and Burdisso 1988).

Meanwhile, Tu et al. (1999) indicated a convergence problem associated with the numerical singularities in the traditional RIA; this impelled the development of a new approach called the performance measure approach (PMA), in which, rather than using the reliability index, an inverse reliability analysis problem is constructed to evaluate the probability performance. Because the numerical singularity exists in only certain numerical extreme cases such as wherein the standard deviation is negligible, it is an insignificant issue in engineering practices. Apart from the numerical singularity associated with the tight standard deviations, the traditional RIA continues to fail to converge under the general setting, a phenomenon that is yet unexplained. This convergence problem of the traditional RIA prompts numerous researchers to select the PMA as a more efficient and robust option for general nonlinear performance functions (Youn et al. 2003, and Youn and Choi 2004).

The RIA and PMA exhibit several important differences. For example, in the RIA, the probabilistic constraints are evaluated based on the most probable point search on the surface of the limit-state function in the uncorrelated standard normal space of random variables (Keshtegar and Miri 2014). In contrast, in the PMA, the probabilistic constraints are evaluated by searching the minimum performance target point on the target reliability surface (Mohsine et al. 2006).

Nevertheless, the RIA and PMA share a common feature. Although they have been applied to various RBDO problems, the related optimizations were based on the failure probability of the structural component. However, as aforementioned, the risk assessment of fatigue-induced sequential failures requires reliability analysis at the system level. The methods of the RIA and PMA exhibit limitations that prevent their application to the problem; moreover, it is necessary to develop a new method to conduct system-reliability-based design optimization (SRBDO) considering the fatigue-induced sequential failures of a complicated structure.

Subsequent studies regarding the problem revealed that for an effective reliability analysis of structures, it is necessary to employ methods assessing the structural reliability at the structural system level rather than based on the reliability analysis of the individual members (Thoft-Christensen and Murotsu 1986). This is because these methods permit the inclusion of the interaction between the structural members into the analysis and design, and also enable the design of structural members considering their status in the structural system. In general, the failure of a member of a statically indeterminate truss structure does not necessarily result in the failure of its structural system.

Another important aspect of SRBDO is that it requires a sophisticated algorithm to effectively solve an optimization problem. Among the various optimization methods developed, the genetic algorithm (GA) is known to efficiently determine an optimized solution by creating generations of design variables repeatedly based on the algorithms of selection, crossover, and mutation. GA has been adapted to structural problems in various engineering disciplines. Murotsu et al. (Murotsu et al. 1988) optimized the size and topology of truss structures using constraints on the failure probabilities of the individual members. Stocki et al. (Stocki et al. 2001) imposed constraints on the values of the componential reliability indices corresponding to the permissible displacements of a few specified nodal points, the permissible stress or local buckling of the structural members, as well as the global loss of stability, to optimize the size and geometry of the truss structures. Kaveh and Kalatjari (Kaveh and Kalatjari 2002, Kaveh and Kalathari 2003, and Kaveh and Kalatjari 2004) employed the force method and GA to optimize the truss structures. Park et al. (Park et al. 2004) proposed a new method for the reliability assessment of structural systems. Togan and Daloglu (Togan and Daloglu 2006) considered the failure probability of a truss structure to be equal to the sum of the failure probabilities of its members, and optimized the cross-sectional areas of the truss members. Kalatjari et al. (Kalatjari et al. 2011) used the algebraic force method and artificial intelligence to assess the reliability of statically indeterminate trusses. Kalatjari and Mansooria (Kalatjari and Mansooria 2011) used the branch-and-bound and the competitive distributed GA methods to optimize the size of the truss structures. Kaveh et al. (Kaveh et al. 2013) proposed certain strategies to improve the accuracy of the reliability analysis of truss structures and to increase the optimization speed of truss structures in opposition to the system reliability constraint. Kim et al. (2013) used the selective search method to identify the dominant failure modes of structural systems. Kaveh et al. (Kaveh et al. 2014) utilized the charged system search (CSS) algorithm as an optimization tool to achieve the minimum reliability index under the limit-state function. Kaveh and Ilchi Ghazaan (Kaveh and Ilchi Ghazaan 2015) utilized certain recent metaheuristic algorithms for the structural reliability assessment.

It is evident from these previous studies that the GA exhibits important advantages to be applied to SRBDO. Firstly, it does not rely on the gradient of the objective function, which can help prevent local optimum solutions and provide the global optimum solution. In addition, the analysis cost of the GA is reasonable. Therefore, in this research, the GA is employed to carry out SRBDO of structural systems. In addition, to consider fatigue-induced sequential failure during SRBDO, the proposed method introduces the Branch-and-Bound method employing system reliability Bounds, termed B^3 method.

2. Proposed Method

2.1. Branch-and-Bound method employing system reliability Bounds: B^3 method

Structural systems which do not have certain level of structural redundancy under the risk of fatigue-induced failure may experience structural collapse from the sequential local failure (Lee & Song 2011, 2012, Karamchandani et al. 1992). Therefore, the identification of the significant sequences of local failures and the quantification of such risks are required for structural design. Several strategies are generally used to identify the failure sequences. One of the strategies is the deterministic search approach (Gharaibeh et al. 2002, Thoft-Christensen and Murotsu 1986). The method using the mean values of the random variables identifies the failure sequence. However, the approach easily culls the critical failure sequences owing to the identified sequences not being guaranteed as critical sequences. Another strategy is the branch-and-bound method (Guenard 1984, Murotsu 1984). Many developed approaching methods have the missing risk despite the efforts to develop the searching method. Another strategy is the branch-and-bound method. After the searching method compares all the failure sequences, the method assumes additional damages for the most likely sequence. The method defines a system failure sequence when the structural collapse is observed at the certain sequence. This searching approach does not miss the critical sequences unlike the previous deterministic search approach and facilitates in reducing the time cost. However, numerous sequences must still be searched to estimate a reliable structural system failure probability. The Branch-and-Bound method employing system reliability Bounds called the B^3 method was recently developed to solve such a challenging structural problem and was applied to various structures (Lee & Song 2011, 2012). To reduce the time cost and to increase the accuracy, disjoint failure sequences are identified to update the upper and lower bounds of the system failure probability. In this study, the B^3 method is described briefly.

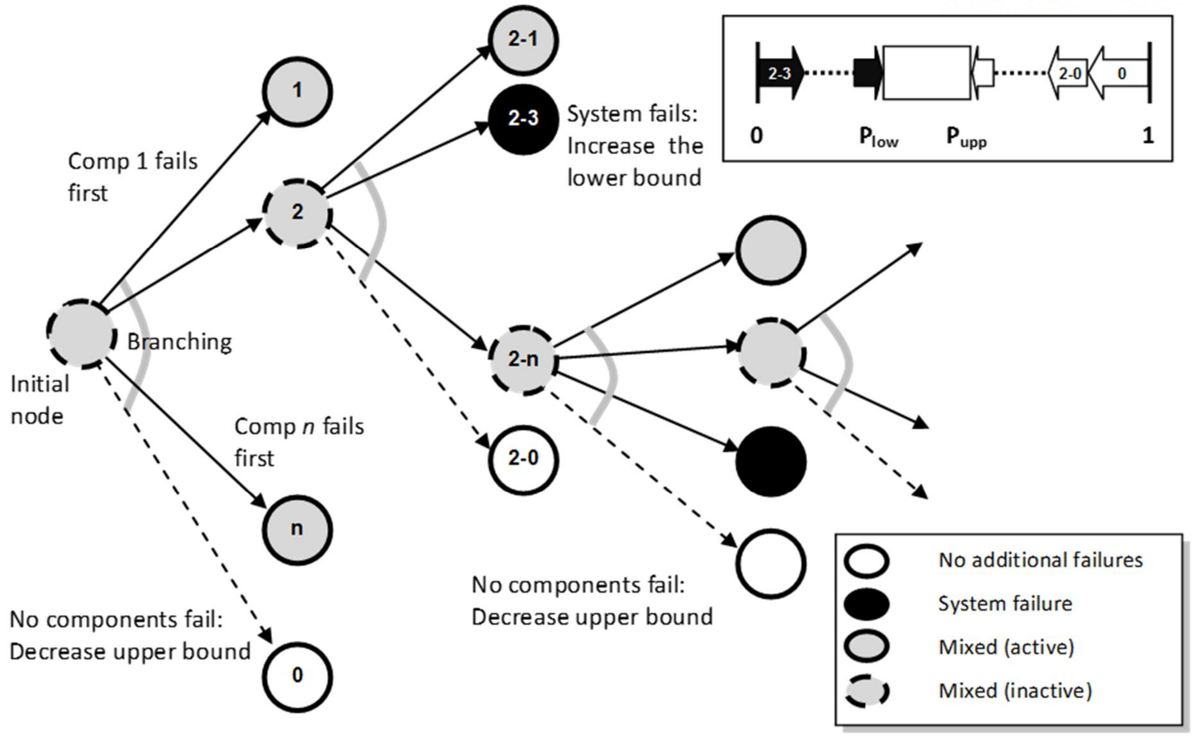

Figure 1. Search procedure of B^3 method

The search process of the B^3 method is illustrated in Figure 1. This iterative procedure leads to a narrower gap between upper and lower bounds. It has been proposed that the procedure be terminated when the gap between the two bounds becomes smaller than 5% of the upper bound, and the application of this procedure to examples of an offshore platform (Lee and Song 2011) and an aircraft longeron (Lee and Song 2012) has shown that the upper bound at the termination point with a 5% gap is very close to the system failure risk obtained from Monte Carlo simulations.

In addition, one important advantage of the B^3 method is that it is very efficient, and hence, the system-level failure probability can be calculated using a relatively small number of structural analyses. This advantage makes it feasible to calculate the sensitivity of the objective function with respect to each design variable, which is required for a design optimization analysis, by simply using finite forward approximation. The proposed search procedure is explained by a flow chart in Figure 2.

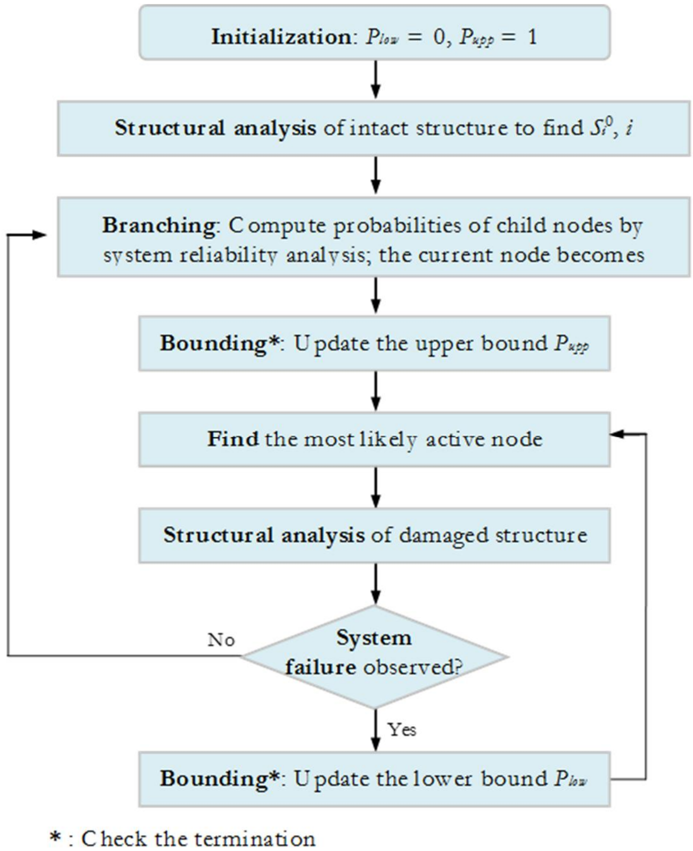

Figure 2. Flowchart of B^3 method

2.2. Genetic algorithm (GA)

The GA was developed by Holland (1992) and Goldberg (1989). The GA is inspired by evolutionary algorithms (Bäck and Schwefel 1993, Schwefel 1993). Mechanisms of the GA are selection, mutations, and crossovers. The global optimum is obtained by the GA logically avoiding the local optima without considering the target reliability surface. However, the GA does not guarantee the optimum solutions even if it provides the proper solutions (Zitzler et al., 2004). Obtaining the global optimum primarily depends on time. As the optimizer approaches the global optimum, it requires more time cost for remarkable improvements. The final improvement of the solution is time consuming (Bäck 1993). For this challenging and time-consuming task, the generation number must be limited to reduce the overall time cost. Thus, several satisfactory solutions may be obtained from the optimized executions (Holland 1992). This is highly effective in exploring the target reliability surface to identify the areas of high potential, but the GA does not determine the optimal solution for all the parameters (Holland 1992). Because the optimization is most sensitive to the mutation operator, the parameters of the mutation operator are determined considering the parallel optimizations (Horton et al., 2017). A value of the population size corresponds to 50 potential solutions in the research, and the iterative process of the optimization was terminated when the best potential solution does not change for 20 generations.

The GA has been studied to optimize various engineering optimization problems, especially in combinatorial problems that require huge computational cost. However, very few cases were investigated for the structural reliability field. The GA determined the maximum reliability solutions to the target material cost of a system for fixed design variables and stated the incremental reductions in the local failure rates and costs (Painton and Campbell 1994, 1995). The algorithm can be reconstructed to optimize the reliability of the mean time to failure (MTTF). The GA is applied to a redundancy-allocation issue with numerous failure modes (Ida et al., 1994). This problem had been studied using the nonlinear method and integer method.

A series-parallel system optimization problem with several subsystems and component selections for each subsystem was analyzed (Coit and Smith 1994). The target reliability space had over 1000 unique solutions, and the GA converged to an acceptable solution. In this study, approximations with neural network were used. It is noteworthy that the exact solutions were not applied. To estimate the global solution effectively, the RBDO method in the system level employing the GA and the B^3 method proposed in this study was used. The GA will be applied to numerical examples for a brief explanation:

In problem 1, the optimization problem refers to the reliability maximization of the series-parallel system in a given redundancy allocation problem. Meanwhile, problem 2 describes the effect of minimizing the cost. These two problems have the same constraints. The GA can be performed under two constraints.

Problem 1

$$\max_{x_1,\ldots,x_s}\left\{\prod_i R_i(x_i \mid k_i)\right\} \tag{1}$$

Subject to

$$\sum_i C_i\left(x_i\right) \le C,\ \sum_i W_i\left(x_i\right) \le W \tag{2}$$

The system reliability can be described as a function of x_{ij} by the following equation:

$$\begin{aligned} R_s\left(x_1,\ldots,x_s | k\right) &= \prod_i R_i(x_i \mid k_i) \\ &= \prod_i\left[1-\sum_{l=0}^{k_i-1}\sum_{t\in\tau}\prod_j binm(t_j, r_{i,j}, x_{i,j})\right] \end{aligned} \tag{3}$$

where τ is such that $\prod_j t_j = l$.

It is impossible to formulate the linear equivalent objective function achieved in the integer formula for this problem. (Ghare and Taylor 1969, Bulfin and Liu 1985, Misra and Sharman 1991, Gen et al. 1993, and Gen et al. 1990). In general, both P1 and P2, i.e., the cost and weight of the system, respectively, are defined as linear functions owing to the assumption that the problem is a reasonable representation of the incremental effect of the cost and weight on the component level. The GA also handles the nonlinear constraints and inseparable constraints of which the integer formulations are not capable.

The size of the target searching space (N) is huge for any size of the problems, even when the size is considered small.

The area of the searching space is also large for any sized problems. Assuming that the number of duplicates has an upper bound, the number of unique system representations is as follows (Feller 1968):

$$N = \prod_i \left[\binom{m_i + n_{max}}{m_i} - \binom{m_i + k_i - 1}{m_i} \right] \tag{4}$$

Furthermore, implementing the GA in a reliability issue can be a challenging task. The implementation process proposed to apply the GA to a target problem having six steps is as follows: 1) solution description; 2) initial population; 3) fitness function; 4) crossover operator; 5) mutation operator; 6) evolution. The procedure is described below:

The GA is a probabilistic optimization algorithm inspired by biological genetics introduced by Holland (Holland 1975) and Goldberg (Goldberg 1989). The procedure of the GA is introduced briefly as follows:

1. Describe the solutions numerically
2. Create an initial generation
3. Select the superior solutions for the next population
   a. Crossover operator
   b. Mutation operator
   c. Discard inferior solutions
4. Repeat process 3 until the termination criteria is satisfied.

Crossover and mutation operators determine the efficiency of the GA. The effect of the crossover operator depends on the convergence rate and the mutation operator helps to avoid the local optimum. The number of next solutions produced in each generation is an adjustable parameter that remains constant during a certain trial.

Typically, the potential solution becomes a binary string for the crossover operator (Holland 1975, and Goldberg 1989). It is more effective than using integer parameters for the combination optimization problem (Antonisse 1989). In the study, this approach is applied. Each set of n_i $(k_i \le n_i \le n_{max})$ is each potential solution to the duplicated assignment problem for each subsystem. n_i components are available for any solution set from among the m_i. The m_i components are arranged in the descending order according to the reliability. The solution vector is represented with $s \cdot n_{max}$ positions. Each subsystem is shown as the n_{max} location that contains the components that are listed in the descending order according to the reliability index. An index of $m_i + 1$ is specified where no additional components are used $(n_i < n_{max})$. The subsystem representations are located close to each other to construct the vector representation.

For the selected p, the initial generation was created by randomly specifying p solution vectors. For a certain subsystem, s integers that are selected randomly represent n_i. Assuming that each possible component set is unlimited, the n_i components were subsequently determined from among the m_i available components. The randomly and uniformly selected components were determined by their reliability. Previous research (Coit and Smith 1994) showed that a population size of 40 yielded the results with low time cost and acceptable solutions. In general, a minimized and efficient population size increases with the problem size.

The target function was a dynamic penalty function determined by the sum of the reliability (P1) or cost (P2) and the degree of its relative suitability. Because the optimum solution is the most efficient and feasible solution. Generally, feasible solutions are the product of viable solutions and breeding, particularly in areas where they are highly unsearchable. A dynamic penalty function based on the square of the constraint violation was defined to provide an efficient search through an unfeasible area and to enable the final best solution. The penalty function used in the research is generated based on a study by (Smith and Tate 1993) and increased the model with the number of occurrences, g. For P2, the objectives and penalty functions used in this analysis are defined in (5). Accurate reliability estimates are used for each k - out - n : G subsystem using (3). The formula for P1 is similar.

$$f(\lambda, v_q) = \left[\sum_i C_i(x_i)\right] + P(\lambda, v_q) \tag{5}$$

An effective solution is provided by exploring the area of the demonstrated sample space by the crossover operator. In this GA, the parent generation was chosen according to the descending order of the target function. A uniform number U selected randomly between 1 and $\sqrt{p}$, and the solution containing the descending order closest to U^2 was chosen as the following selection procedure according to Tate and Smith (1994). The crossover operator maintained the same genetic information from the parent and subsequently randomly selected from both parents at the same probability for the different components. Owing to the descending order of the reliability of the solutions, they were often matched. To analyze the combination problems, this crossover operator is modified to be superior to the traditional crossover algorithm (Syswerda 1989).

In the mutation process, the deformation occurred randomly on the selected solutions. For each GA process, a predetermined number of mutations are established within a generation. Each parameter of the selected solution vector in the mutation process was changed with the probability as the amount of mutation rate. A modified component transforms to the index of $m_i + 1$ with 50% probability, and to be one of the m_i options that randomly selected the component from them, with 50% probability.

In GA, the survival of the fittest is applied. After the processes associated with creating a generation such as crossover and mutation, the best solutions, p from the previous generation and the new generations were created for the subsequent generation. The value of fitness is quantified by the fitness function. After the selection process, the mutation operator was performed. In the mutation process, the best solutions are excluded to compare with the best solutions from the previous generation and current generation to select the best solutions from all the generations. This is an entire process of the elitist selection (Ida et al. 1994). The GA stops when it creates a sufficient number of generations, although the optimal solution was obtained.

2.3. Proposed method for SRBDO

In this research, a new method for the SRBDO employing the B^3 method and GA is proposed. As described in sections. 2.1 and 2.2, both the B^3 method and the GA require an iterative procedure. In the proposed SRBDO method, the GA repeatedly requires the execution of the B^3 analysis to evaluate the constraints of the system failure probability related to fatigue-induced sequential failures. The outline of the proposed method is illustrated in the following figure:

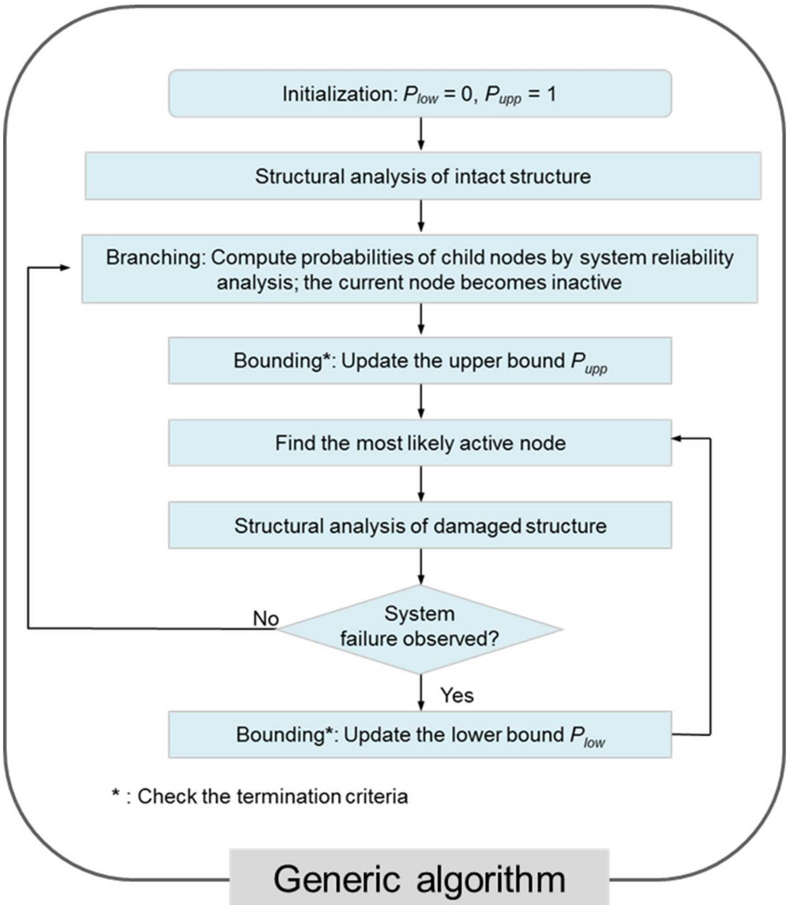

Figure 3. Outline of proposed SRBDO method

In the first step of the GA analysis, the initial population for the design variables is generated to be the first chromosome. The B^3 analysis is performed with the initial population to calculate the related system failure probabilities, verify whether the specified constraints are satisfied, and evaluate the fitness function. Subsequently, the next generation chromosomes are generated with the crossover of a breeding operator, a mutation operator, and an elite selector. The B^3 analysis is performed and the system failure probability is calculated with each generation of chromosome. For example, if ten chromosomes exist in a generation, the B^3 analysis is conducted ten times. Hence, the GA continues to generate chromosomes with a higher fit, and the GA analysis is terminated when any of the following criteria is satisfied:

1) The mean of the fitness function values in a generation is lower than a specified value ε_1.
2) During N_{GA} generations, the optimum solution does not change, and the average difference in the chromosomes in a generation is lower than a specified value ε_2.

The specified constraints should be satisfied in any of the cases above. In the numerical example of this research, ε_1 and ε_2 are both assumed to be 0.01, and N_{GA} is specified as 10.

3. Numerical Example: Multi-layer Daniels System

3.1. Problem description

After the reliability of a group of ideally brittle wires which are assumed to have a deterministic and identical elastic moduli exposed to a deterministic load was investigated by Daniels (1945), the example called the Daniels system was introduced as a numerical example to develop and test new probability methods in various studies (Lee and Song 2012, Kang et al. 2012, Straub and Der Kiureghian 2007, Song and Der Kiureghian 2003, Gharaibeh et al. 2002).

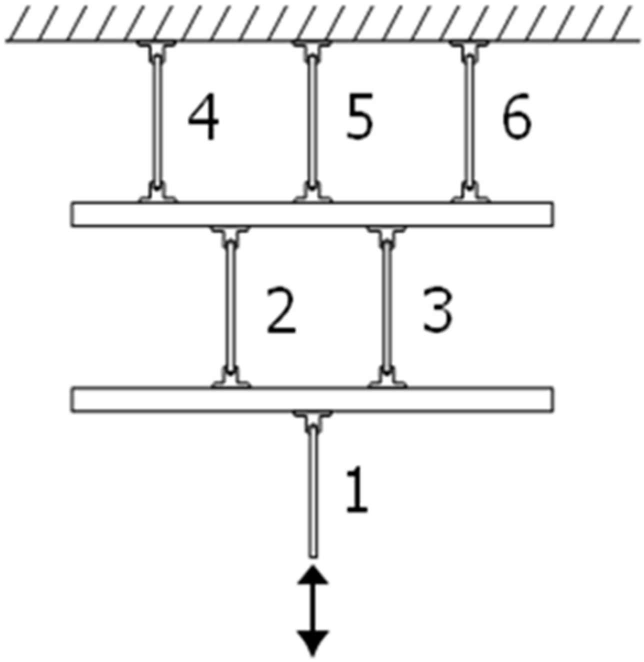

Figure 4. Multi-layer Daniels system (Lee and Song 2012)

As shown in Figure 4, the multi-layer Daniels system consists of six wires that are assumed to have a deterministic and identical elastic moduli and be perfectly brittle, as assumed in the conventional Daniels system problem. The widths of the six bars are assumed to be W_1 = 38.1 mm, $W_2 = W_3$ = 19.05 mm, and $W_4 = W_5 = W_6$ = 12.7 mm, and their cross-sectional areas are assumed to be A_1 = 100 mm^2, $A_2 = A_3$ = 50 mm^2, and $A_4 = A_5 = A_6$ = 33.33 mm^2. In this example, the uncertainties of material parameters C and m in the Paris equation (i.e., Equation 1), the initial crack length a_i^0 and external load I are introduced as random variables, and the statistical properties of the random variables considered are summarized in Table 1.

Random variables	Mean	c.o.v.	Distribution type	Number of random variables
C	1.36×10^{-13}	0.533	Lognormal	6
m	3.0	0.02	Lognormal	6
a^0 (mm)	0.11	1.0	Exponential	6
I (KN)	17.2	0.1	Lognormal	1

Table 1. Statistical properties of random variables

For simplicity, all the random variables are assumed to be statistically independent of each other. In addition, the following deterministic parameters are used: loading frequency (ν_0) = 100 (cycles/h); inspection cycle (T_s) = 2000 h; and critical crack lengths are assumed to be 80% of the widths (i.e., W_i's, $i = 1,\ldots,6$), which implies that a member will fail when the related crack size attains 80% of the member width.

The SRBDO problem in this research can be designed as follows:

$$\begin{aligned} &\min_{\mathbf{d}} f\left(\mathbf{d}, \mathbf{X}\right) \\ \text{s.t. } &P\left[g_i\left(\mathbf{d}, \mathbf{X}\right) \le 0\right] \le P_i^t,\ i = 1, \ldots, n \\ &P_{sys} \le P_{sys}^t \end{aligned} \tag{6}$$

where d is the vector of the deterministic design variables (such as the section area); $\mathbf{X}$ is the vector of the random variables; $f(\cdot)$ is the objective function; $g_i(\cdot)$, $i = 1,\ldots,n$ is the i-th limit-state function representing the member failure; $Psys$ is the system failure probability; P_i^t is the target failure probability of the structural members; and P_{sys}^t is the target failure probability of the structural system.

However, if the sum of the areas of the six members is to be minimized while maintaining the system failure probability below a certain level, it is an SRBDO problem, and the member areas are considered to be design variables. In this study, two SRBDO problems are introduced to test the proposed method. In Problem #1, the multi-layer Daniels system has to be designed such that the sum of the areas of the six members is minimized and the system failure probability is less than 5×10^{-3}. In Problem #2, a constraint in addition to that for Problem #1 is added: the probability of the most critical failure sequence should be less than 1×10^{-3}. For these two optimization problems, the abovementioned member areas (i.e., 100 mm^2 for member 1, 50 mm^2 for members 2 and 3, and 33 mm^2 for members 4, 5, and 6) are used as the initial values for the optimization, and the other conditions are assumed to be identical.

3.2. Analysis results

First, the B^3 method is applied to the multi-layer Daniels system with specified areas. Initially, the upper and lower bounds on the system failure probability are specified as one and zero, respectively; moreover, there is only one node, which implies the intact structure in the event tree. After the first branching, the event tree is expanded as shown in Figure 5.

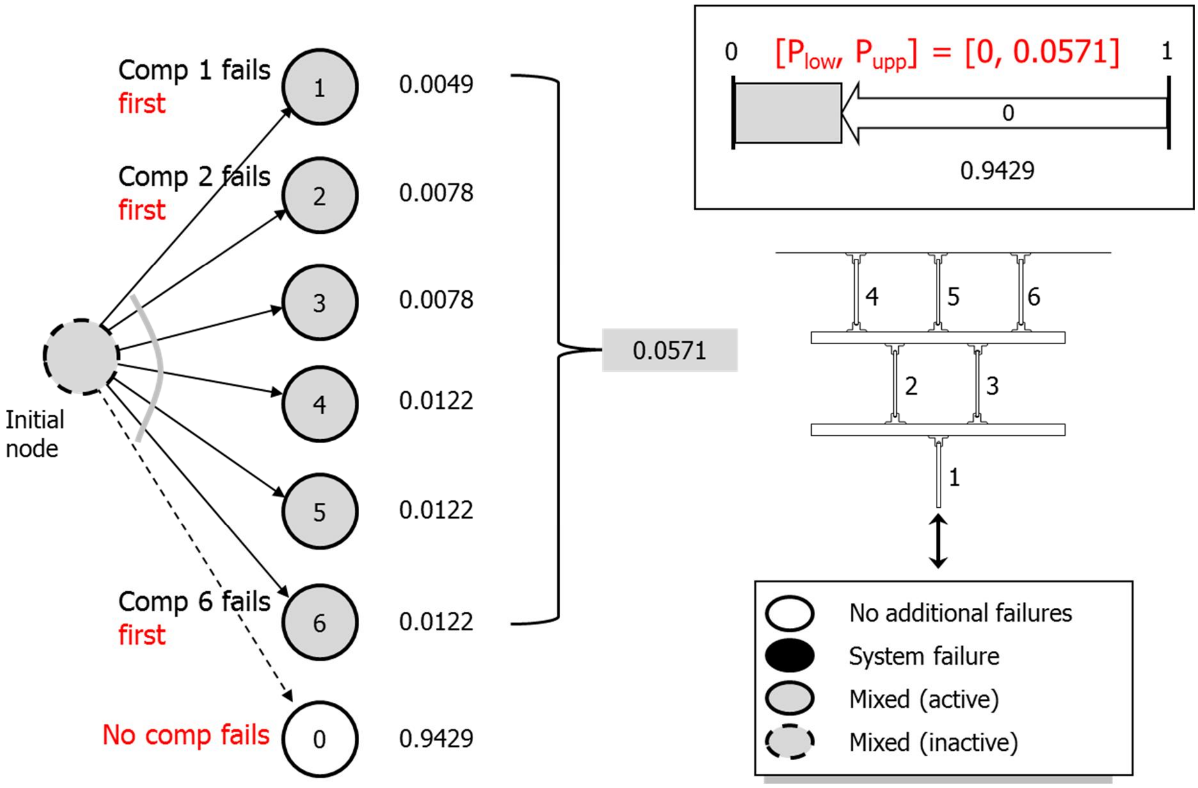

Figure 5. Results after first branching for initial design

In Figure 5, grey nodes 1 through 6 and one white node 0 are branched out of the initial node, and the sum of the seven nodes is verified to be one. Because node 0 indicates a non-system-failure case, the upper bound on the system failure probability can be reduced by the probability of node 0 and is obtained as 0.0571.

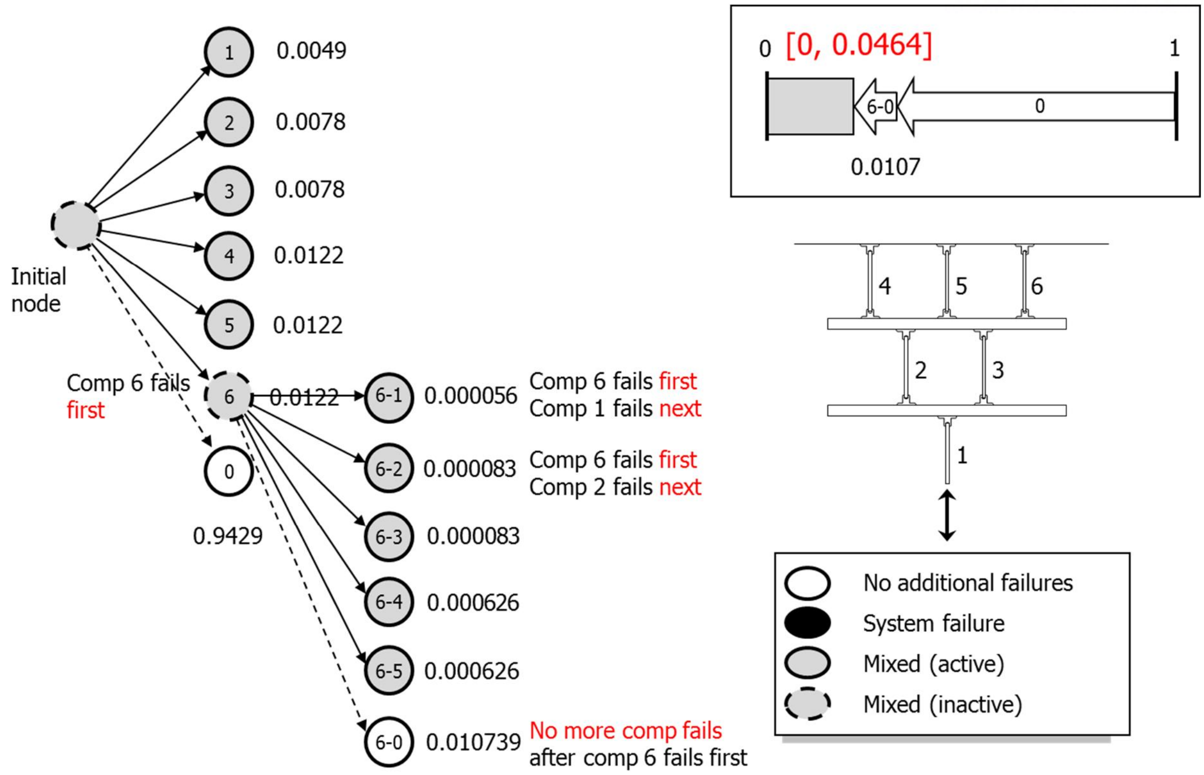

Figure 6. Results after second branching for initial design

For the subsequent branching, a node with the maximum probability among the available grey nodes is selected (in this case, node 6), and it is verified whether the associated structure will fail or not. Because the structural system can survive even after member 6 fails, further branching is performed out of the node, as shown in Figure 6. From the branching, five grey nodes and one white node is obtained. Because the white node (i.e., 6-0) indicates another non-system-failure case, the upper bound on the system failure probability can be reduced again by the probability of the white node. As a result, the upper bound is calculated to be 0.0464 in this step.

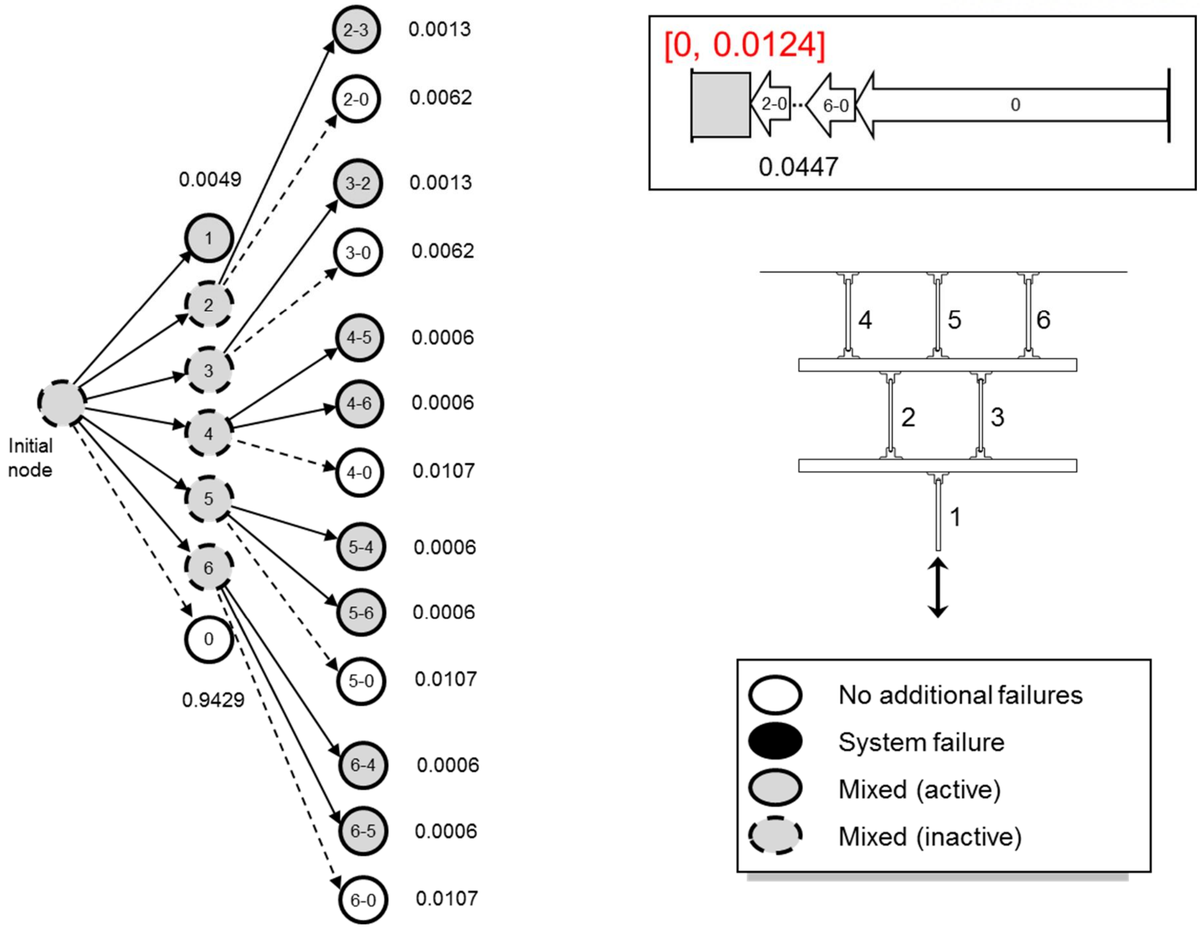

Figure 7. Results after sixth branching for initial design

After doing four more times of branching the corresponding structural analysis, the event tree is expanded the one shown in Figure 7. In the figure, for the sake of space, only some of the active nodes which have relatively higher probabilities than the others are presented. As node 1 has the highest probabilities, the subsequent branching starts from it. However, it is obvious that failure of member 1 results in the system failure. Therefore, node 1 is identified as a system failure case, and the lower bound is increased by the probability of node 1; the upper and lower bounds are estimated to be [0.0049, 0.0458] in this step (see Figure 8).

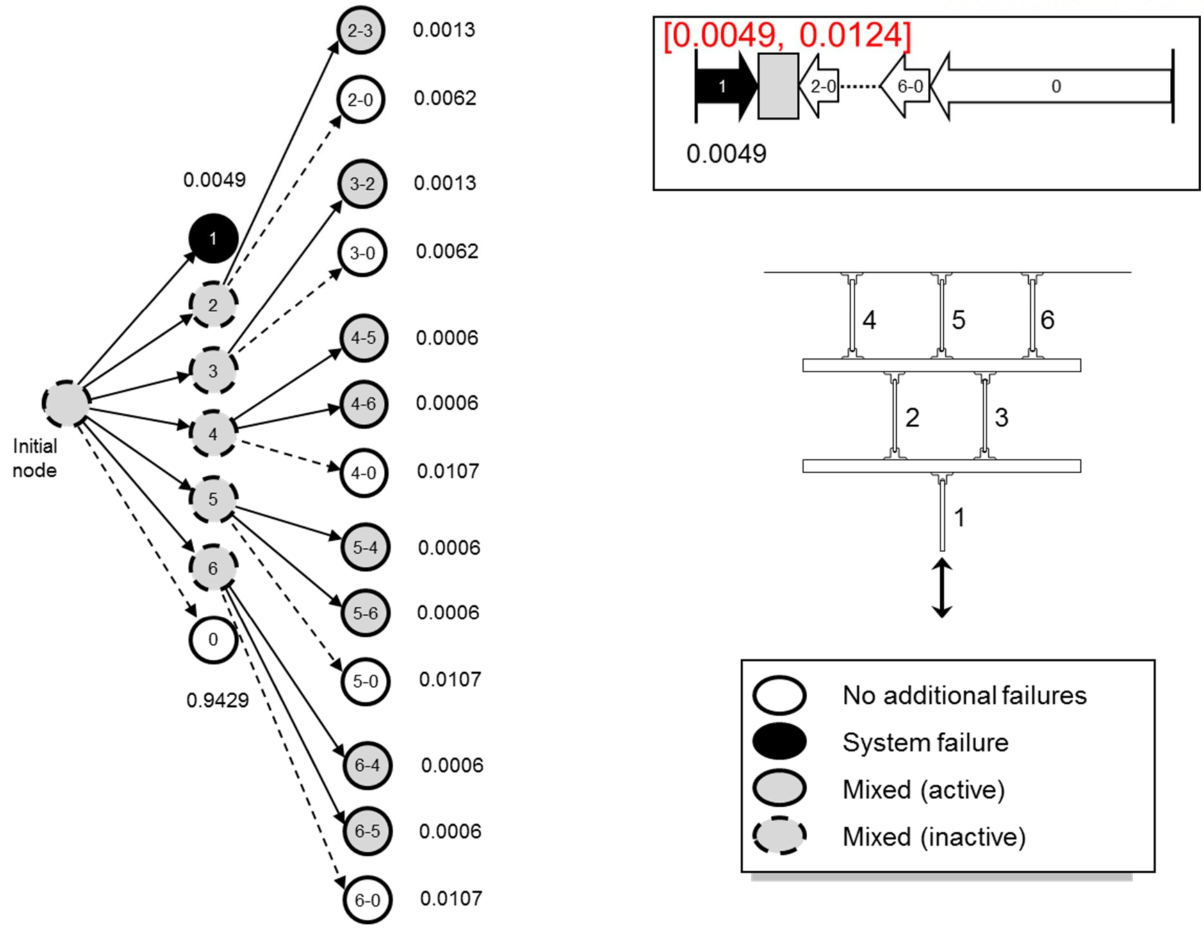

Figure 8. Results after seventh branching for initial design

It is noticeable that each structural analysis results in either the case identification of a system failure or another branching, which contributes to update the system probability bounds, by increasing the lower bound or decreasing the upper bound. That is, during the B^3-based search, each structural analysis contributes to the narrowing the system failure probability bounds. In this example, the iterative process is stopped when the difference between the bounds becomes less than the five percentage of the upper bound. Figure 9 shows how the bounds are updated with the increasing number of “structural analyses” during the B^3 search.

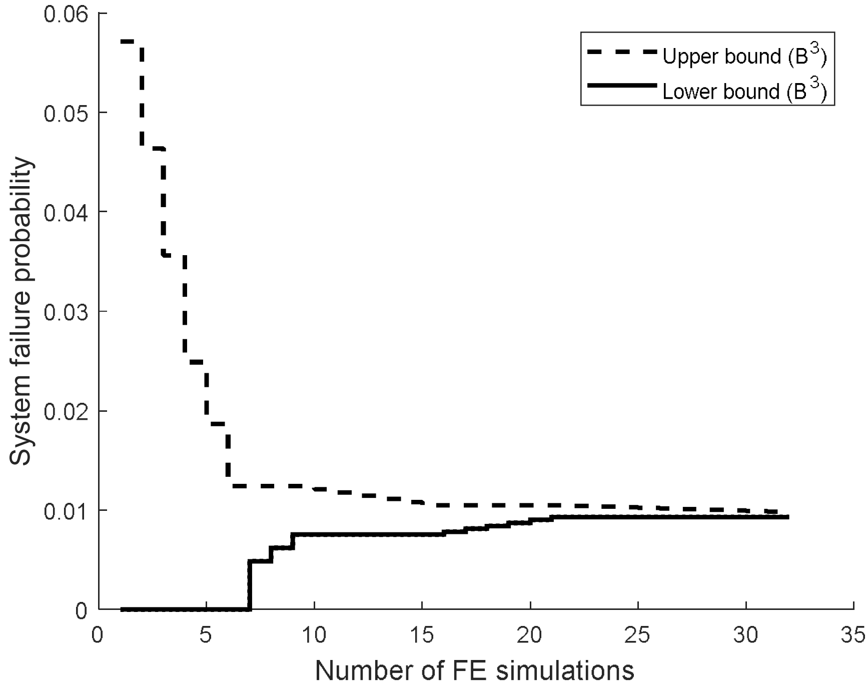

Figure 9. Upper and lower bounds of the system failure probabilities for initial design

As shown in Figure 5, the lower and upper bounds are calculated from the B^3 analysis to be as 9.767×10^{-3} and 9.289×10^{-3}, respectively, and a total of 32 structural analyses were performed during the analysis. Based on previous studies on the B^3 method, it is known that the upper bound is very close to the true solution at the moment when the B^3 analysis is terminated (Lee and Song 2011, 2012). Accordingly, the upper bound is returned to the GA as the system failure probability.

In addition, the B^3 analysis identifies the critical failure sequences in the decreasing order of their likelihood. In this example, the most critical failure sequence was "1," and its probability was estimated to be (i.e., member 1 fails first) 4.871×10^{-3}. Figure 10 and Table 2 show all of the the critical failure sequences and their probabilities identified from the B^3 analysis.

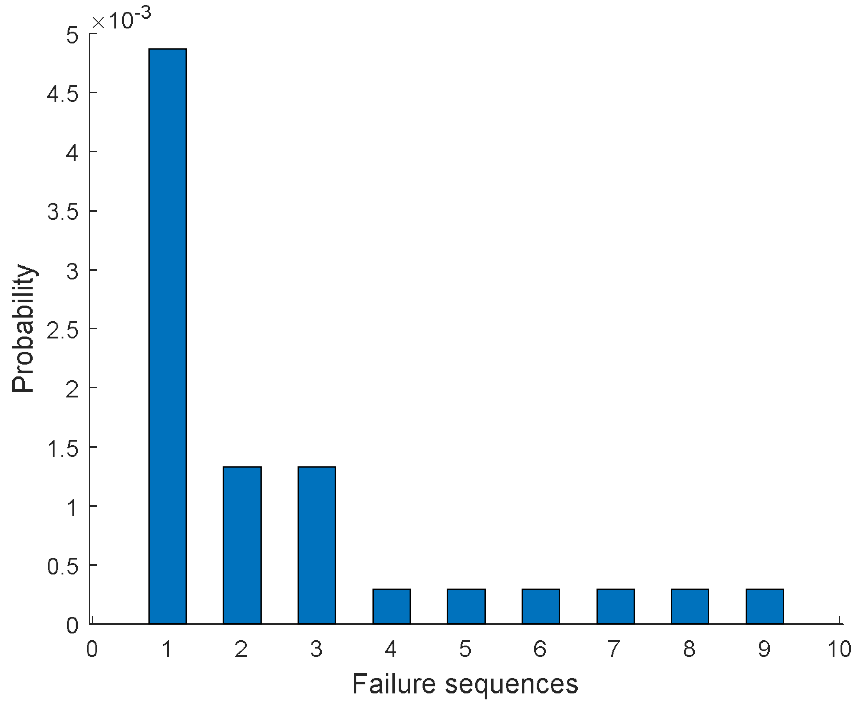

Figure 10. Identified critical failure sequences and their probabilities for initial design

Failure sequence	Probability by B^3 method ($\times 10^{-3}$)
1	4.871
$2 \rightarrow 3$	1.332
$3 \rightarrow 2$	1.332
$4 \rightarrow 5 \rightarrow 6$	0.294
$4 \rightarrow 6 \rightarrow 5$	0.294
$5 \rightarrow 4 \rightarrow 6$	0.294
$5 \rightarrow 6 \rightarrow 4$	0.294
$6 \rightarrow 4 \rightarrow 5$	0.294
$6 \rightarrow 5 \rightarrow 4$	0.294

Table 2. Identified critical system failure sequences and probabilities for initial design

However, these analysis results were obtained with the initial values of the design variables (i.e., 100 mm^2 for member 1, 50 mm^2 for members 2 and 3, and 33 mm^2 for members 4, 5, and 6). If the sum of the areas of the six members is to be minimized while maintaining the system failure probability below a certain level, it is an SRBDO problem, and the member areas are considered as design variables.

In Problem #1, the multi-layer Daniels system has to be designed such that the sum of the areas of the six members is minimized and the system failure probability is less than 5×10^{-3}. The initially assumed values of the member areas are used as the initial values for the optimization, and the other conditions are assumed to be identical. As a result, the optimized areas are obtained as $A_1 = 107.9$ mm^2, $A_2 = A_3 = 52.1$ mm^2, and $A_4 = A_5 = A_6 = 33.8$ mm^2 from the proposed method, and the sum of these areas is calculated to be 313.5 mm^2. Compared with their initial values, it is noticeable that the areas are increased overall, and the structural system is likely to become more robust than before. To verify this, the B^3 method analysis is performed with the optimized values.

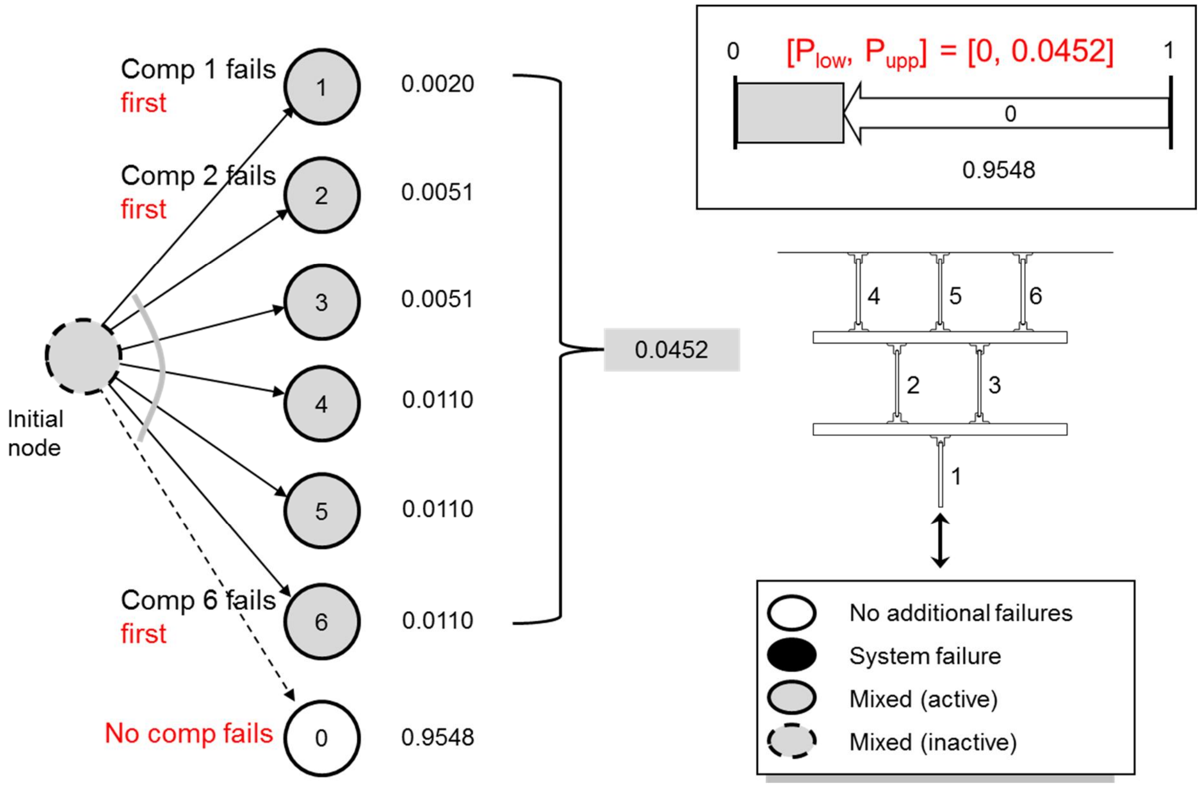

Figure 11. Results after first branching for Problem #1

First, Figure 11 shows the analysis results after the first branching with the optimized area values, for Problem #1. It is noteworthy that compared with Figure 5, the probabilities of the grey nodes reduce, whereas the white node probability increases. This indicates that the structural system was designed to exhibit a lesser failure probability than that of its initial design.

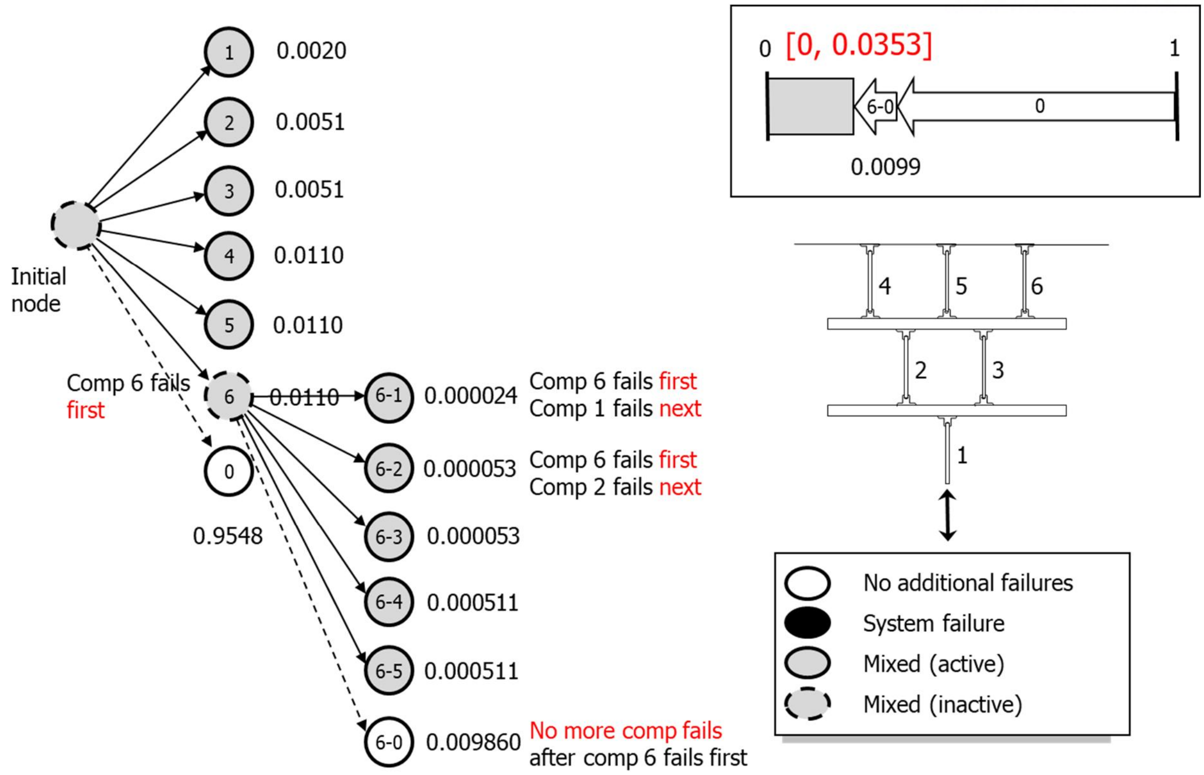

Figure 12. Results after second branching for Problem #1

Figure 12 shows the analysis results after the second branching with the optimized area values, for Problem #1. It is noteworthy that compared with Figure 6, the probabilities of the grey nodes reduce, also the white node probability decreases due to the probability of node 0. The probability of white node 0 increases by 0.0119 over the differences of 0.0049 between the system probability of initial design and the system probability of design #1.

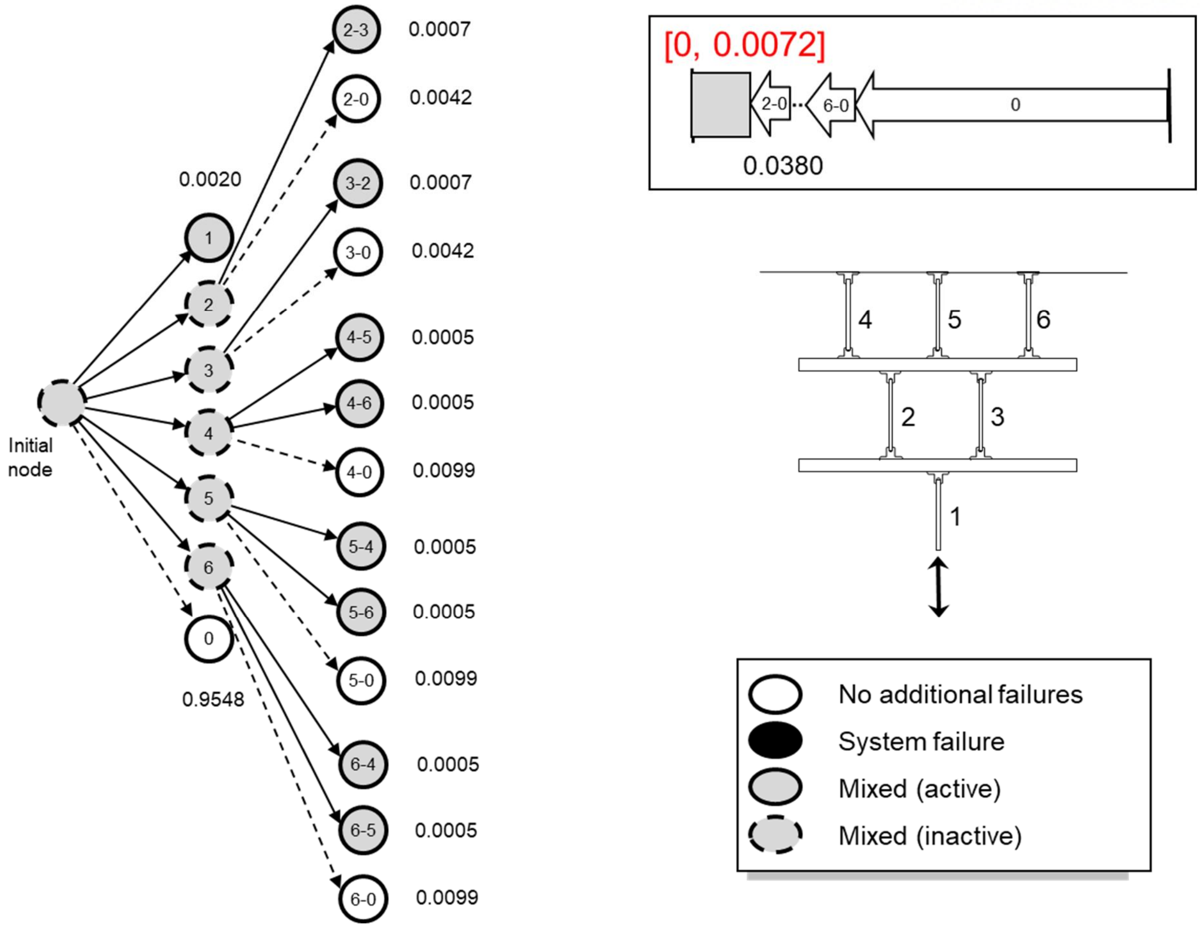

Figure 13. Results after sixth branching for Problem #1

Figure 13 shows the analysis results after the sixth branching with the optimized area values, for Problem #1. Comparing Figure 13 with Figure 6, the probabilities of the grey nodes decrease and the probabilities of the white nodes reduce for the same cause as in previous Figure 12.

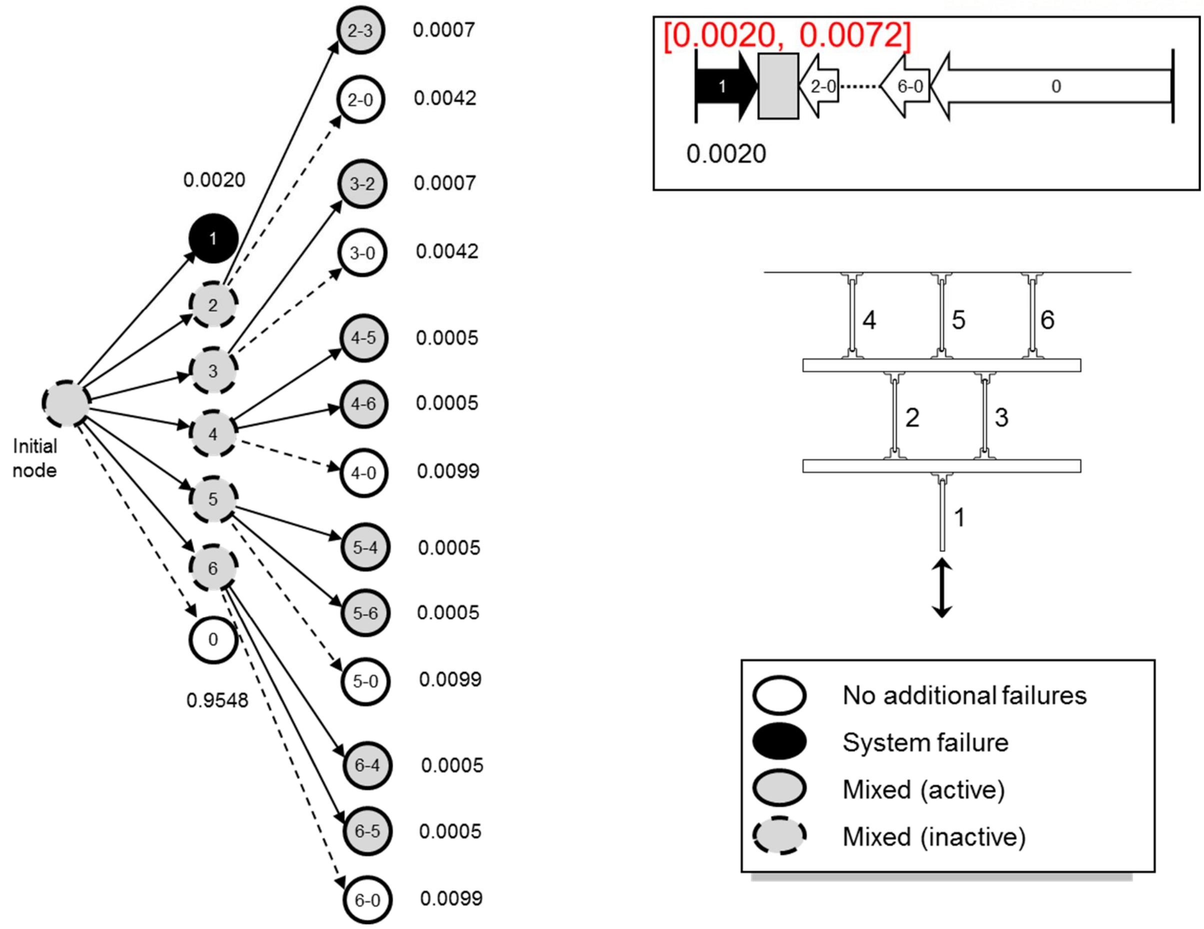

Figure 14. Results after seventh branching for Problem #1

Figure 14 shows the analysis results after the seventh branching. The figure shows that overall, the probabilities of the white nodes (indicating non-system-failure cases) are larger than those with the initial values of the design variables shown in Figure 8. However, the probability of the black node (indicating a system failure case) is decreased. These also establish that the structural system was designed to exhibit a lower failure probability than that of its initial design. The final analysis results of the upper and lower bounds and critical failure sequences are shown in Figures 15 and 16 and Table 3.

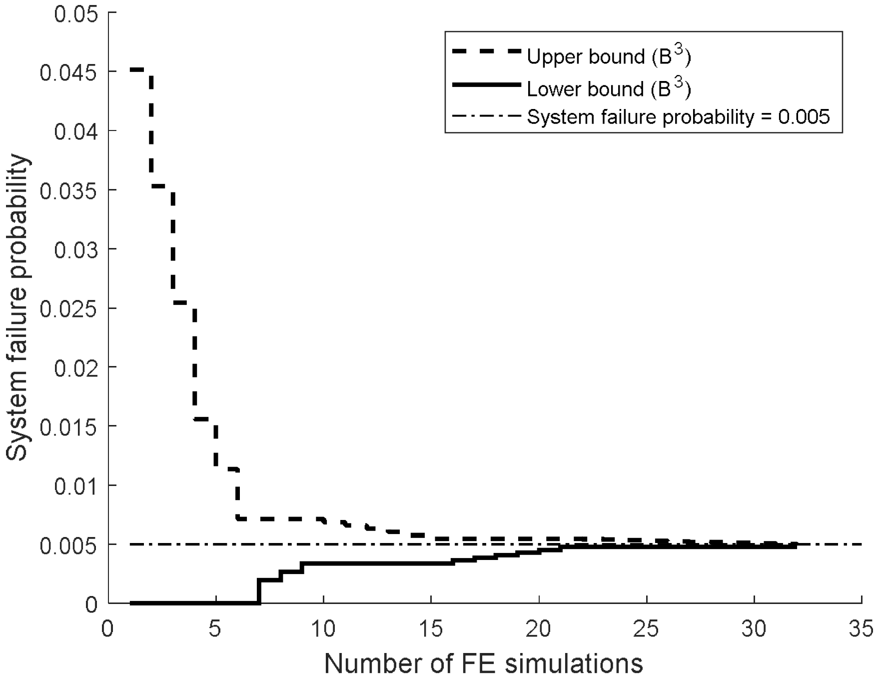

Figure 15. Upper and lower bounds of system failure probabilities for Problem #1

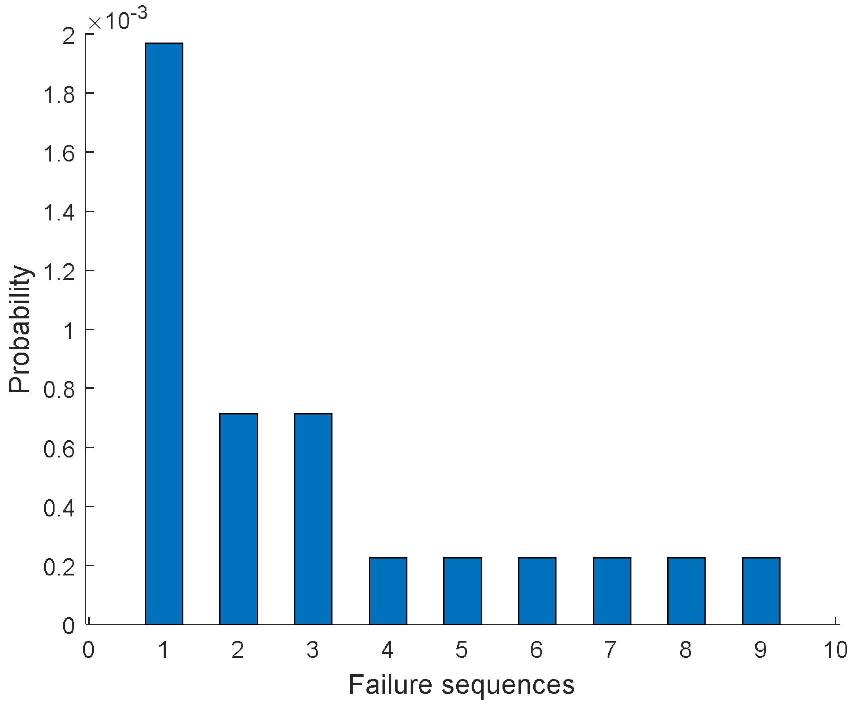

Figure 16. Identified critical failure sequences and their probabilities for Problem #1

Failure sequence	Probability by B^3 method ($\times10^{-3}$)
1	1.969
2 → 3	0.715
3 → 2	0.715
4 → 5 → 6	0.227
4 → 6 → 5	0.227
5 → 4 → 6	0.227
5 → 6 → 4	0.227
6 → 4 → 5	0.227
6 → 5 → 4	0.227

Table 3. Identified critical system failure sequences and probabilities for Problem #1

Figure 15 establishes that the structure is designed such that the system failure probability is lower than the specified threshold (i.e., 0.005), and the critical failure sequence probabilities become less than those in Figure 10. Apparently, these imply that the structure is designed to be more robust with the increased member areas.

As another SRBDO problem, in Problem #2, a constraint apart from that for Problem #1 is added: the probability of the most critical failure sequence should be less than 1×10^{-3}. Similar to Problem # 1, for these two optimization problems, the initially assumed values of the member areas (i.e., 100 mm^2 for member 1, 50 mm^2 for members 2 and 3, and 33 mm^2 for members 4, 5, and 6) are used as the initial values for the optimization, and the other conditions are assumed to be identical. The optimized areas are obtained to be A_1 = 113.8 mm^2, $A_2 = A_3$ = 51.8 mm^2, and $A_4 = A_5 = A_6$ = 32.8 mm^2 from the proposed method, and the sum of these areas is calculated to be 316.1 mm^2. It is noticeable that in this case too, compared with their initial values, the areas are increased; moreover, the structural system is likely to become more robust than before. In addition, it is observed that the area of member 1 is increased more than those of the other members. This is because the most critical failure sequence was identified by “1” (i.e., member 1 fails first); therefore, the member area needs to be increased more than the other member areas to reduce the probabilities of the failure-sequences related to member 1. To verify these, the B^3 method analysis is performed with the optimized values.

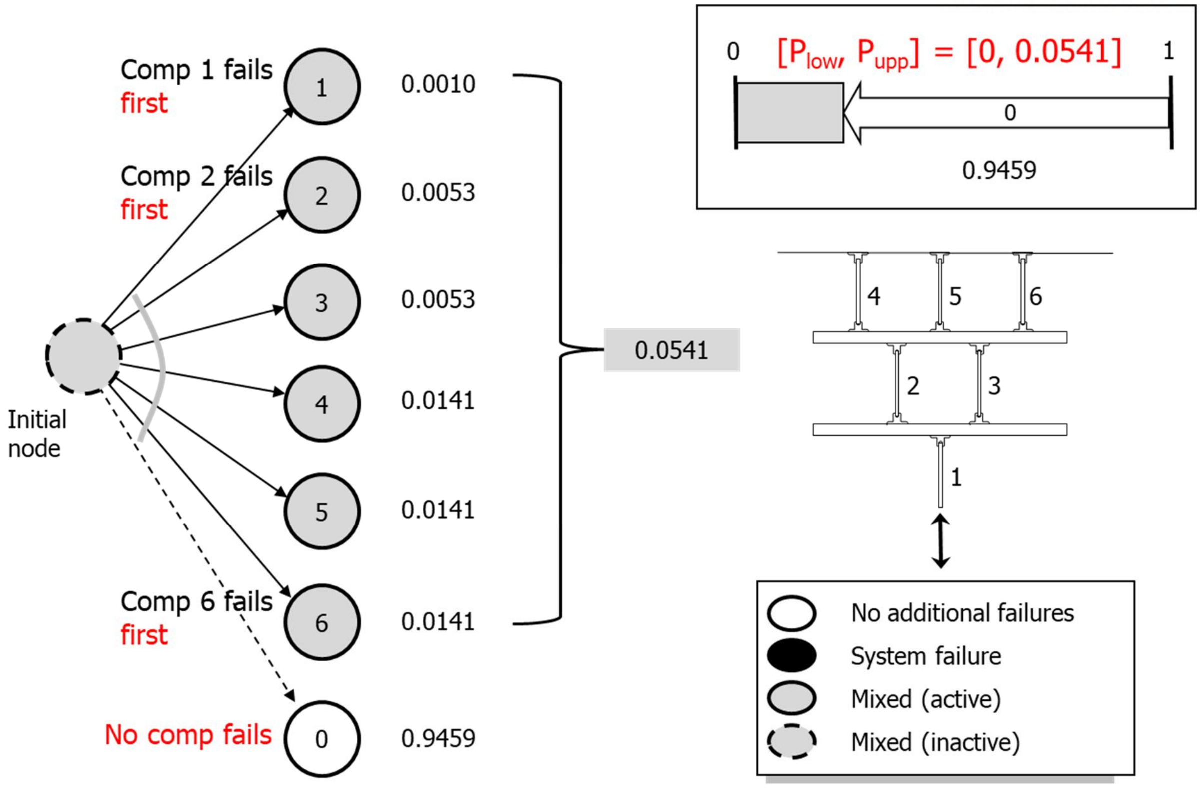

Figure 17. Results after first branching for Problem #2

First, Figure 17 shows the analysis results after the first branching with the optimized area values, for Problem #2. It is noteworthy that compared with Figure 11, the probabilities of the grey nodes except node 1 increase, whereas the probabilities of the white node and grey node 1 decrease. This is the effect of the changed design variables. Compared with design #1, component areas from 2 to 6 for design #2 are decrease and the area of component 1 increases.

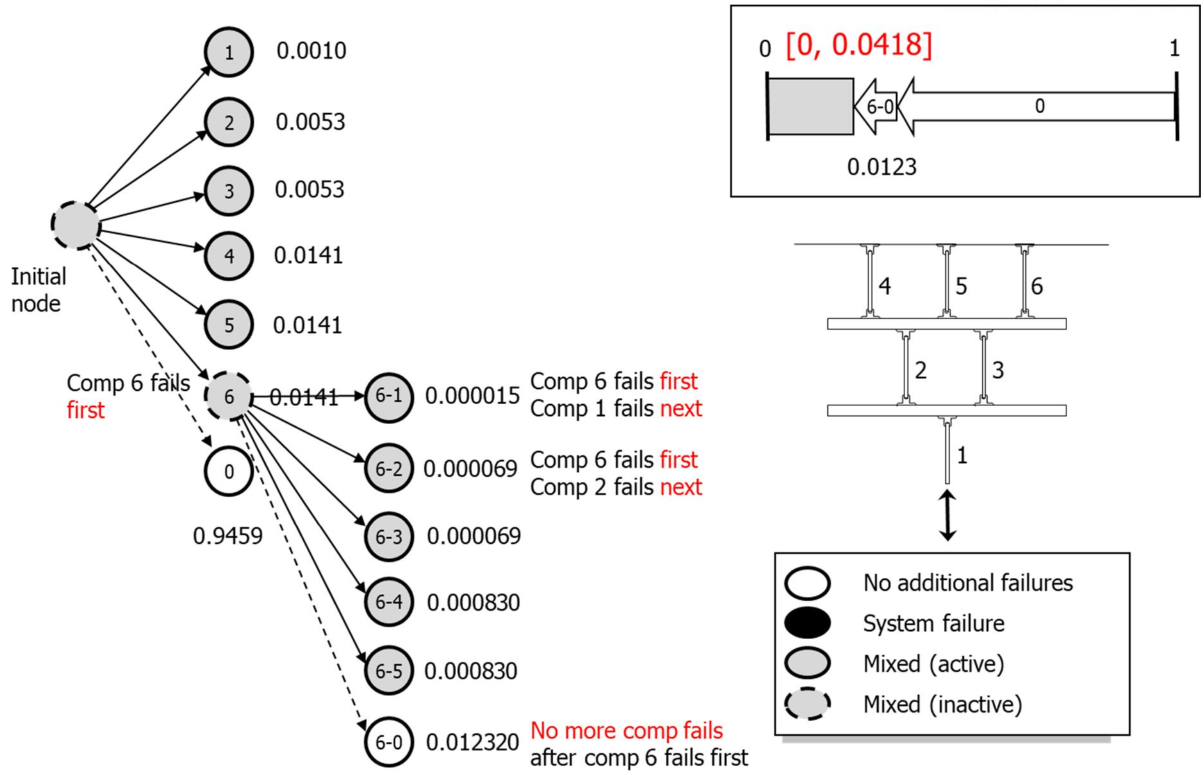

Figure 18. Results after second branching for Problem #2

Figure 18 shows the analysis results after the second branching with the optimized area values, for Problem #1. It is noteworthy that compared with Figure 12, the probabilities of the children nodes branched out of the node 6 increase except node 6-1. Because the design #2 has a constraint for a most critical sequence probability should be smaller than 0.005, only the area of the component 1 increases as the others decrease.

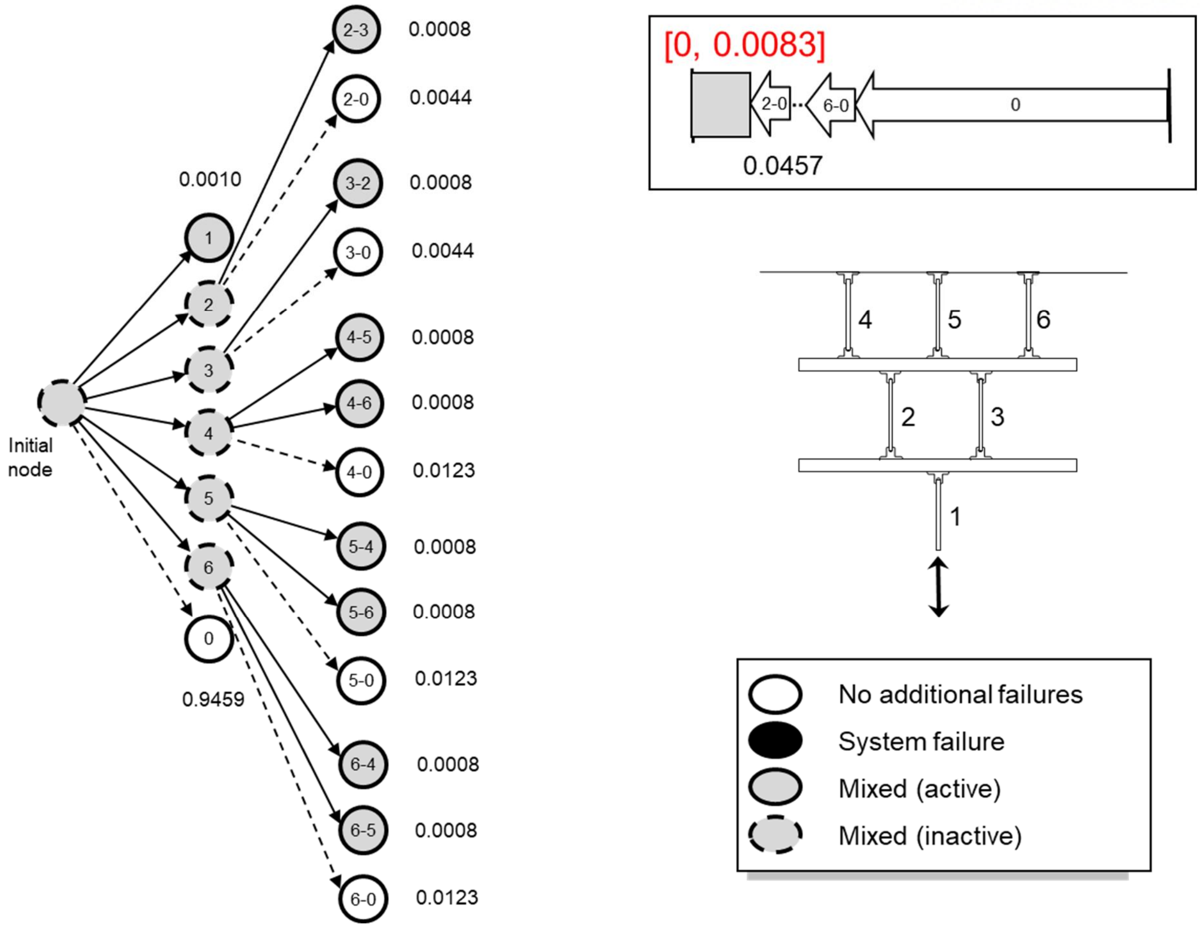

Figure 19. Results after sixth branching for Problem #2

Figure 19 shows the analysis results after the sixth branching with the optimized area values, for Problem #1. Comparing Figure 19 with Figure 13, there was an effect of the changed variable node 1, the upper bound of the system is similar (0.0083 and 0.0072 for design #2 and design #1 respectively).

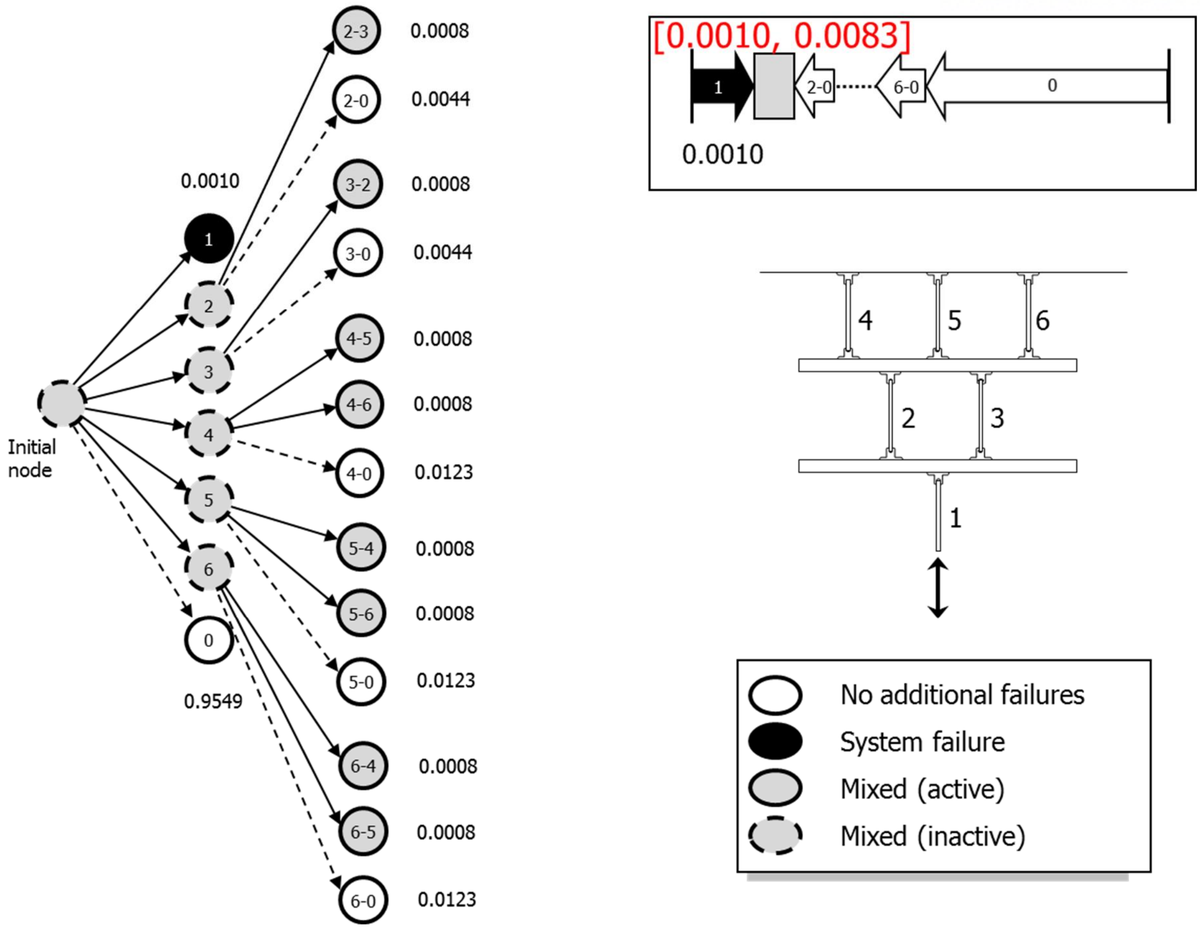

Figure 20. Results after seventh branching for Problem #2

Figure 20 shows the analysis results after the seventh branching. It is noteworthy that in the figure, the probabilities of the white nodes (indicating non-system-failure cases) are larger overall than those with the initial values of the design variables shown in Figures 8 and 11. In addition, the failure probability of node 1 is decreased to be the specified threshold (i.e., 0.001). The final analysis results of the upper and lower bounds and the critical failure sequences are shown in Figures 20 and 21 and Table 4.

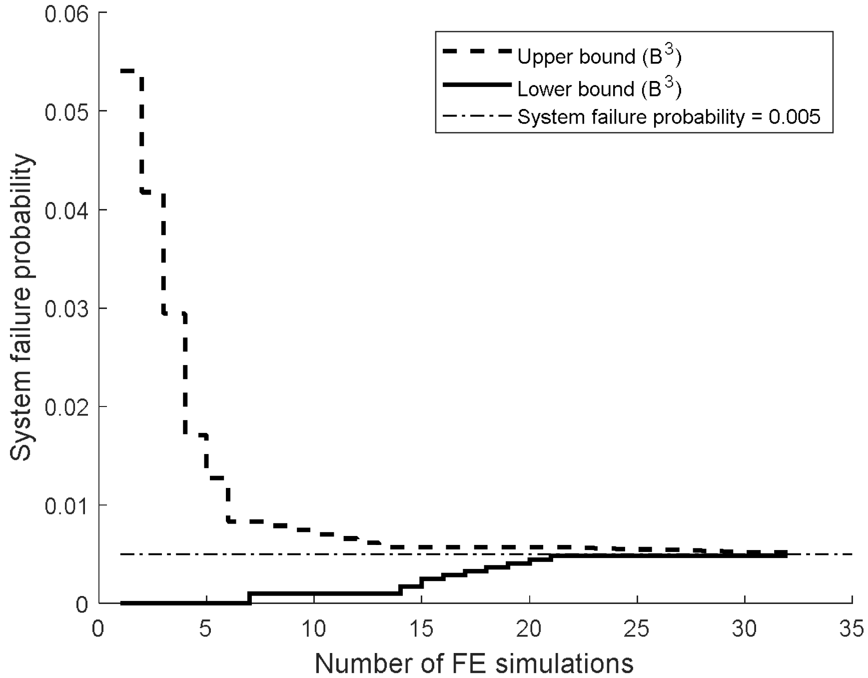

Figure 21. Upper and lower bounds of system failure probabilities for Problem #2

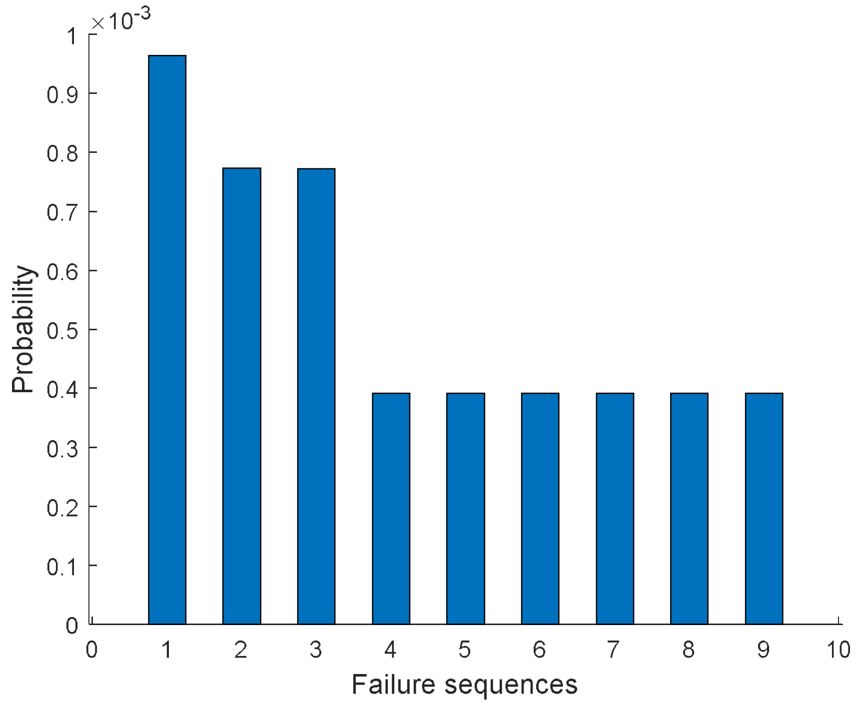

Figure 22. Identified critical failure sequences and their probabilities for Problem #2

Failure sequence	Probability by B³ method ($\times 10^{-3}$)
1	0.964
2 → 3	0.773
3 → 2	0.773
4 → 5 → 6	0.392
4 → 6 → 5	0.392
5 → 4 → 6	0.392
5 → 6 → 4	0.392
6 → 4 → 5	0.392
6 → 5 → 4	0.392

Table 4. Identified critical system failure sequences and probabilities for Problem #2

Figure 21 verifies that the structure is designed such that the system failure probability is lower than the specified threshold (i.e., 5×10^{-3}), and Figure 22 shows that the failure probability of node 1 is decreased to be the specified threshold (i.e., 1×10^{-3}). For a more effective comparison of the analyses, the initial member areas and those obtained for Problems #1 and #2 are summarized in Table 5.

Problem	Area (mm²)							System failure probability ($\times 10^{-3}$)	Most critical sequence probability ($\times 10^{-3}$)
	A_1	A_2	A_3	A_4	A_5	A_6	Total area		
Initial	100	50	50	33.33	33.33	33.33	300	9.767	4.871
#1	107.9	52.1	52.1	33.8	33.8	33.8	313.5	4.996	1.969
#2	113.8	51.8	51.8	32.8	32.8	32.8	316.1	4.989	0.966

Table 5. Analysis results for Problems #1 and #2

First, the member areas obtained for Problem #1 increase compared to their original areas, and the total area also increases from 300 mm² to 313.5 mm². Consequently, the system failure probability is estimated to be 4.996×10^{-3}, which satisfies the specified constraint. The most critical failure sequence and its probability are determined to be “1” and 1.969×10^{-3}, respectively.

However, for Problem #2, the probability of the most critical failure sequence should be less than 1×10^{-3}. As a result, the total area increases again. In addition, the area of member 1 (i.e., A_1) increases significantly although the areas of the other members marginally decrease. This is because the failure sequence “1” continues to be identified as the most critical failure sequence, and consequently, the probability is significantly reduced.

Furthermore, an important advantage of the proposed method is that it can perform the SRBDO analysis at a reasonable cost. In this numerical example, the required number of the B³ analysis was 32 for each of Problems #1 and #2. Considering that approximately 3 min is required for each B^3 analysis with a generic personal computer (2.90 GHz CPU and 8.00 GB RAM), it requires several hours to solve the specified SRBDO problems. For future research, the proposed method will be applied to more realistic structural problems such as offshore platform and steel bride. Another research topic is that more advanced optimization algorithms will be introduced to find a global optimum solution more effectively.

4. Conclusions

This paper proposes a new approach for the RBDO of structural systems considering structural redundancy against fatigue-induced failure. To properly consider fatigue-induced sequential failure at a system level, the proposed approach employs the B^3 method and calculates system-level probabilities and sensitivities that are required for the RBDO of structural systems. The proposed approach is tested by applying it to a numerical example of a multi-layer Daniels system. First, the structural system with the initial design (of 300 mm^2 total area) was analyzed by the B^3 method, and the probabilities of system failure and the most critical failure path were determined to be 9.767×10^{-3} and 4.871×10^{-3}, respectively. However, in Problem #1, the system failure probability was constrained to be less than 5×10^{-3}; moreover, based on the proposed method, the total area was increased to 313.5 mm^2. In this case, the probabilities of system failure and the most critical failure path were determined to be 4.996×10^{-3} and 1.969×10^{-3}, respectively. In Problem #2, the most critical failure path probability was constrained to be less than 1×10^{-3}; moreover, based on the proposed method, the total area was increased to 316.1 mm^2. In this case, the probabilities of system failure and the most critical failure path were determined to be 4.989×10^{-3} and 0.966×10^{-3}, respectively. In both Problems #1 and #2, the specified constraints were satisfied. The analysis results for numerical examples are summarized as follows:

1) The system failure probability is by performing 32 steps to identify only 9 cases of system failure sequences in 146 possible system failure sequences.

2) The total area is increased as the system failure probability is decreased.

3) Problem 2 has more limitations (1 more constraint) than problem 1, but the costs of the two problems are similar. (number of steps are same)

4) By limiting the failure probability of each member, the area of the member with the highest node failure probability is increased most, and the total area is slightly increased.

Therefore, it has been successfully demonstrated that the proposed approach is effective for determining the system-level risk of the fatigue-induced failure of a structure and performing SRBDO analysis.

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