Analytic Radiation Models of a Microstrip Line on a Slotted Ground Plane

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Abstract

In this Letter, analytic models for electromagnetic (EM) radiation analysis of a microstrip line (trace) on a slotted ground plane are proposed. The EM radiation is separated by the effects of trace and slot, and analytic radiation models for each effect are presented. The radiated electric field in the far region is calculated at a frequency up to 10 GHz, and the effect of the slot size on the radiation is examined. The proposed models are in good agreement with the numerical results while requiring a negligible computation time.

Key Words: Common-Mode Current, Ground Plane, Microstrip Line, Printed Circuit Boards, Radiation, Slot.

I. INTRODUCTION

As electronic devices operate at higher frequencies, concerns about unwanted electromagnetic (EM) radiation from printed circuit boards (PCBs) are also increasing. When a high-speed signal passes over a slotted ground plane, the radiation, as well as the crosstalk between signals, increases. This is a typical PCB-level EM interference problem that has been studied for a long time. In [1], crosstalk and radiation that occur when a signal line passes through a split reference plane in a high-speed multilayer PCB were analyzed using a slot waveguide model. Recently, an EM radiation model of a stripline with a slotted ground plane using a partial-element equivalent-circuit method was proposed [2]. These methods focused on the crosstalk between signals and applied numerical methods to calculate the radiation.

In this letter, analytic models for EM radiation analysis of a microstrip line on a slotted ground plane are proposed. Using these models, the radiation mechanism can be intuitively understood, and accuracy similar to that of a numerical simulation can be obtained by a quick calculation.

II. ANALYTICAL FORMULATION

Fig. 1 shows the structure of a microstrip line (trace) passing over a slotted ground plane. The radiated fields can be calculated by dividing region (I) over the microstrip line and region (II) over the ground plane. In this paper, as shown in Fig. 2, the trace and slot radiation models were separately derived and calculated. Therefore, the radiated field in region (I) can be expressed as the sum of the trace and slot radiations, and the radiated field in region (II) can be expressed by the slot radiation as follows:

$$E_\phi^I = E_\phi^t + E_\phi^s \tag{1}$$

$$E_\phi^{II} = E_\phi^s. \tag{2}$$

The magnetic fields are obtained by $H_\theta^{I,II} = E_\phi^{I,II}/\eta_0$, where η_0 is the intrinsic impedance of free space.

First, when a microstrip line is terminated with a resistor equal to its characteristic impedance, the radiated electric field in the far region by the microstrip line can be expressed as follows [3]:
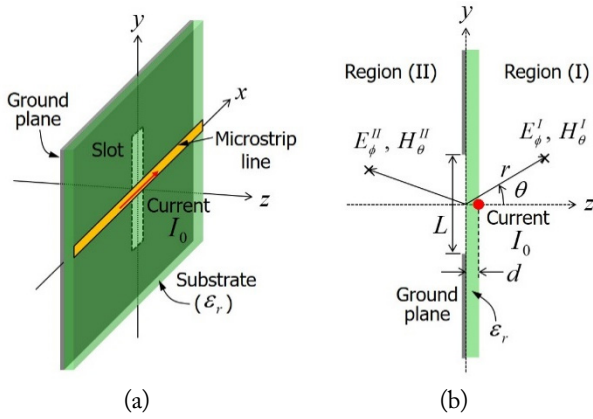
Fig. 1. Problem of electromagnetic radiation from a microstrip line on a slotted ground plane: (a) perspective view and (b) cross-sectional view.
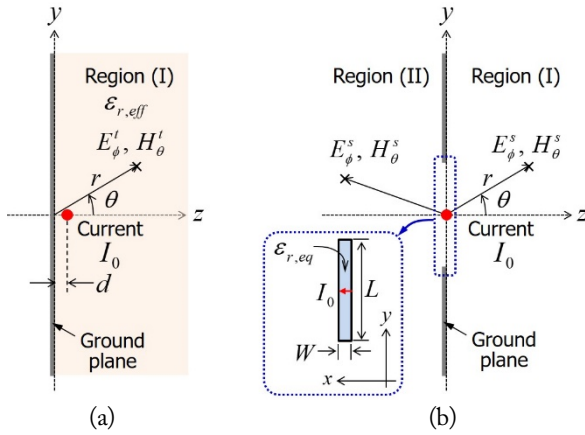

Fig. 2. Equivalent problem: (a) trace radiation model and (b) slot radiation model.

$$E_\phi^t(r,\theta,\phi) = -\frac{j\omega\mu_0 e^{-jk_0 r}}{2\pi r} I_0 d \cos\theta \sin\phi \frac{(1-e^{-jk_0\alpha l})}{\alpha}, \quad (3)$$

where I_0 is the current flowing along the trace, l is the length of the trace, d is the thickness of the dielectric substrate, $\alpha = \sqrt{\epsilon_{r,eff}} - \sin\theta\cos\phi$, $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the wave number of free space, and ω is the angular frequency. $\epsilon_{r,eff}$ indicates the effective dielectric constant when region (I) is homogeneously filled with the same dielectric material, and it is calculated using the dielectric constant ϵ_r of the substrate [4], as follows:

$$\epsilon_{r,eff} = \left(\frac{\epsilon_r+1}{2}\right) + \left(\frac{\epsilon_r-1}{2}\right)/\sqrt{1+\frac{12d}{tw}} \quad (4)$$

where tw is the width of the trace.

Next, let us look at the slot radiation model. As shown in Fig. 2(b), a long slot is $L \times W$ of the length and width, and it lies long along the y-axis. When the direction of the microstrip line and the slot are perpendicular to each other, EM coupling becomes the maximum and then EM radiation by the slot is the greatest. Recently, the radiated field from a long slot induced by

an infinite line current was calculated using an equivalent magnetic dipole moment on the slot [5]. However, this method is inaccurate in the microstrip structure, where the line current and the slot on a ground plane are remarkably close to each other. Therefore, we propose a slot antenna model excited by a current source, which is the microstrip line current, to be directly applied to the slot, as depicted in Fig. 2(b). In the radiation model of a slot antenna, a voltage source is typically applied to the center of the slot [6]. However, this study employs a current source. The radiation equation of the slot antenna is modified as:

$$E_\phi^s(r,\theta,\phi) = -\frac{je^{-jk_0 r}}{\pi r} I_0 Z_{sl} \left(\frac{\cos(\sin\theta \cdot k_{eq}L/2) - \cos(k_{eq}L/2)}{\cos\theta \sin(k_{eq}L/2)}\right), \quad (5)$$

where $k_{eq} = \omega\sqrt{\mu_0\epsilon_0\epsilon_{r,eq}}$. As depicted in Fig. 2(b), we assume the slot is filled with a homogeneous medium with the equivalent dielectric constant $\epsilon_{r,eq}$, which is defined by averaging the effective dielectric constant of region (I) and the air dielectric constant of region (II) as $\epsilon_{r,eq} = (\epsilon_{r,eff}+1)/2$. Z_{sl} is the slot impedance given by [7], which is expressed as:

$$Z_{sl} = \frac{j}{2}Z_{os}\tan(k_{eq}L/2) \quad (6)$$

where Z_{os} is the characteristic impedance of the coplanar strip line [4, 7].

III. ANALYSIS RESULTS AND VERIFICATION

To verify the accuracy of the proposed radiation models, E_ϕ^t and E_ϕ^s were calculated for the slots with two varied sizes, and E_ϕ^I and E_ϕ^{II} were also calculated using Eqs. (1) and (2). Then, they were compared with the numerical results using CST Microwave Studio [8]. The parameters were set as $d = 1$ mm, $tw = 1$ mm, $l = 125$ mm, $\epsilon_r = 4.3$, and $I_0 = 1$ A. The PCB size was assumed 200×150 mm. In Figs. 3 and 4, the radiated electric fields were calculated at the position of $r = 3$ m on the $\pm$ z-axis. When the slot size ($L = 20$ mm, $W = 1$ mm) is small (case 1), the trace radiation is larger than the slot radiation. However, when the slot size ($L = 80$ mm, $W = 4$ mm) is large (case 2), the slot radiation is dominant. Above 1 GHz, the slot radiation prevailed regardless of its size and had peaks at its resonant frequencies. In Fig. 5, the radiated electric field patterns at 0.5 GHz on the yz plane were also calculated as a function of θ. Compared with the numerical results (CST), the differences are less than 3 dB at $\theta \le 60°$ for E_ϕ^I and $\theta \ge 120°$ for E_ϕ^{II}. The analytic models of E_ϕ^I and E_ϕ^{II} agree well with the numerical results for all cases.

Table 1 shows the computation time required to obtain the results in Fig. 4. When using an ordinary PC (Intel Core i5-7500 and 8-GB memory), the computation time of the analytic
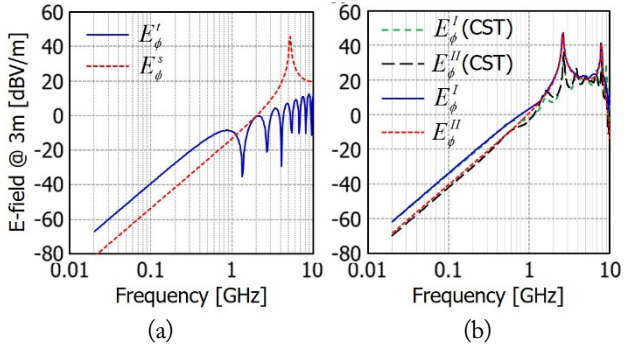
Fig. 3. Radiated electric fields of case 1: (a) trace and slot radiations and (b) total radiations in regions (I) and (II).
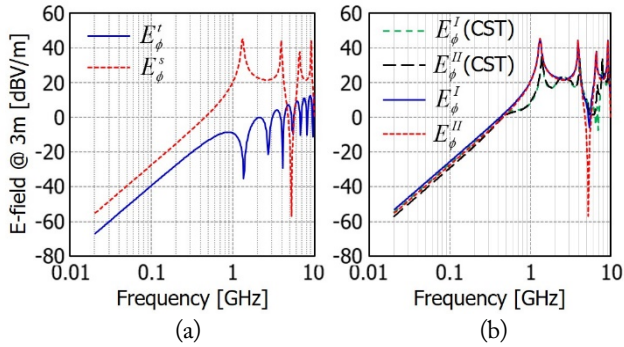

Fig. 4. Radiated electric fields of case 2: (a) trace and slot radiations and (b) total radiations in regions (I) and (II).
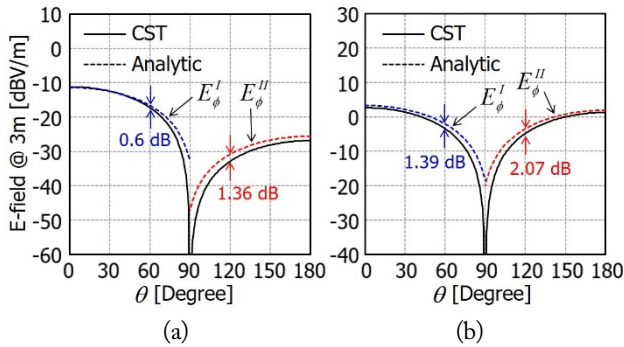

Fig. 5. Radiated electric field patterns at 0.5 GHz on yz plane: (a) case 1 and (b) case 2.

Table 1. Comparison of computation times for Fig. 4

	Computation time
Analytic models	1.51 sec
Numerical method (CST)	13 min 45 sec

models is less than 2 seconds, while that of the numerical simulation is more than 13 minutes. This implies that the proposed analytic models are computationally efficient, as well as accurate.

IV. CONCLUSION

Analytic models for the radiation analysis of microstrip lines on a slotted ground plane were proposed. The radiation was separated by the effects of trace and slot, and the radiated electric field in the far region was examined in terms of the slot size up to 10 GHz. The proposed models were in good agreement with the numerical results while requiring a negligible computation time.

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