# Imaging through Random Media Using Coherent Averaging

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A new phase retrieval method for imaging through random media is proposed and demonstrated. Although methods to recover the Fourier amplitude through random distortions are well established, recovery of the Fourier phase has been a more difficult problem and is still a very active and important research area. Here, it is shown that by simply ensemble averaging shift-corrected images, the Fourier phase of an object obscured by random distortions can be accurately retrieved up to the diffraction limit. The method is simple, fast, does not have any optimization parameters, and does not require prior knowledge or assumptions about the sample. The feasibility and robustness of the method are demonstrated by realizing all computational diffraction-limited imaging through atmospheric turbulence as well as imaging through multiple scattering media.

### 1. Introduction

Imaging through random media is of paramount importance in many imaging scenarios where the acquired images are fundamentally distorted prior to acquisition, such as in imaging through atmospheric and oceanic turbulence, geophysical or biological media, and non-line-of-sight imaging.<sup>[1]</sup> To achieve this challenging feat, advances in hardware-based adaptive optics have shown great potential in recovering diffraction-limited resolution through dynamic disturbances.<sup>[2]</sup> However, hardwarebased adaptive optics has strict requirements that must be fulfilled to guarantee optimal performance. Since the wavefront distortion must be known prior to correction, an effective guide star has to be available to measure the wavefront distortions and the total wavefront correction time has to fall safely within the decorrelation time of the random media. In addition, for efficient wavefront correction through highly scattering media such as thick biological tissue, the number of independent modes that must be corrected for satisfactory performance increases linearly with the signal level enhancement making fast wavefront correction technologically challenging.<sup>[3]</sup>

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To overcome such difficulties, computational image recovery using the information hidden in the distorted images have been demonstrated to be a powerful alternative in the field of astronomy known as speckle interferometry.<sup>[4]</sup> Here, only the statistical properties of random distortions are used and there is no need to find and correct the exact wavefront distortions which greatly relaxes the complexity and constraints on the experimental system. In astronomical speckle interferometry, image recovery using distorted images is generally divided into two steps, (1) recovering the Fourier amplitude and (2) recovering the Fourier phase of the hidden object. To obtain the

Fourier amplitude, Labeyrie first realized that short instants of turbulence do not erase the high spatial frequency information of an object but rather distorts this information.<sup>[4b]</sup> By simply taking the ensemble average of the Fourier transform of the autocorrelation of multiple randomly distorted short exposure images, Labeyrie showed that we can retrieve the power spectrum of the original object as follows,

$$\left\langle \left| \tilde{I}_{n}(u,v) \right|^{2} \right\rangle = \left| \tilde{O}(u,v) \right|^{2} \cdot \left\langle \left| \tilde{P}_{n}(u,v) \right|^{2} \right\rangle$$
(1)

where  $I_n$  is the *n*th acquired distorted image, Ois the object of interest and  $P_n$  is the *n*th distorted point spread function (PSF) due to both the random distortion and intrinsic imaging system parameters, and tilde stands for the Fourier transform. In contrast, if we take the ensemble average of the Fourier transform of the short exposure images we obtain,

$$\left\langle \tilde{I}_{n}(u,v)\right\rangle = \tilde{O}(u,v) \cdot \left\langle \tilde{P}_{n}(u,v)\right\rangle$$
<sup>(2)</sup>

Equation (2) is just equivalent to taking the Fourier transform of a long exposure image where  $\langle \tilde{P}_n(u, v) \rangle$  reaches zero for angles greater than  $\frac{r_0}{\lambda}$ . Here,  $r_0$  is the Fried parameter which describes the characteristic length (or the effective aperture) over which the incident wavefront can be considered to be uniform.<sup>[5]</sup> Labeyrie's key insight was that by simply taking the absolute square in the frequency domain prior to ensemble averaging,  $\langle |\tilde{P}_n(u, v)|^2 \rangle$  is now non-zero up to the diffraction limit and enables recovery of the power spectrum of the object.<sup>[4b,c,5,6]</sup> Due to the simplicity and robustness, Labeyrie's method has become a popular general method that is currently widely used to recover the Fourier power spectrum of an object through random disturbance.

However, recovery of the object requires not only the Fourier amplitude (square root of Fourier power spectrum) but also information about the Fourier phase. In this second step of speckle interferometry, variants of the Fienup type iterative phase retrieval methods have been widely used to recover the Fourier phase from only the Fourier power spectrum information.<sup>[7]</sup> The algorithm consists of iteratively imposing spatial domain constraints such as non-negativity, known supporting regions of the object, or both, while using the recovered Fourier amplitude as the Fourier domain constraint. However, iterative phase retrieval algorithms need to solve a non-convex problem and are known to be very sensitive to almost all algorithm parameters such as prior assumed constraints, initial starting guesses, and input-output scaling factors. In practice, many different initial guesses and/or input-output factors are often tested empirically until a satisfactory image is recovered (total number of initial guesses and parameter tuning trials required for successful phase retrieval differ widely for each experiment. For example, see Figures S1 and S2, Supporting Information). This brings limitations in computation speed and restricts establishing a single unique algorithm with fixed parameters that works robustly for all types of data generated in various experimental scenarios.

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To surmount such problems, we can think about directly obtaining more information from the distorted images in addition to the Fourier power spectra. As these distorted images are still "images," they naturally contain information about the Fourier phase as well. To obtain undistorted Fourier phase information, ensemble averaging the bispectrum of the distorted images has been shown to be an effective method.<sup>[8]</sup> The bispectrum is defined as the Fourier transform of the triple correlation. While the Fourier phase is lost in autocorrelation, the triple correlation retains the phase information. Furthermore, when the coherent transfer function C(u, v) of the random turbulence is a stationary random variable and the real and imaginary parts have zeromean Gaussian distribution, the bispectrum is real and non-zero up to the diffraction limit and the Fourier phase of the object of interest can be obtained from the phase of the ensemble average of the bispectrum of distorted images.<sup>[8b]</sup> In combination with the Fourier power spectrum retrieved from the autocorrelation of the distorted images in Equation (1), the original object can be recovered. However, as the bispectrum of a signal doubles its dimensionality, the bispectrum of a 2-D image is 4-D imposing high demands on computational memory. The object phase recovery from the bispectrum phase is also computationally expensive. To deal with such high computational loads, signal processing techniques using the properties of the bispectrum<sup>[9]</sup> or accelerating computation through graphics processing unit based parallelization have been demonstrated.<sup>[10]</sup> However, widespread use of the bispectrum for phase retrieval is still largely limited due to the excessive computational load, slow speed, and the related difficulties in application to large images.

In this work, we propose a new framework for phase retrieval in imaging through random media. Similar to the bispectrum analysis, the Fourier phase of an object is fully retrieved solely from the information contained in the randomly distorted images. Inspired by Labeyrie's method and the shift-and-add approach in astronomy<sup>[11]</sup> we demonstrate that ensemble averaging the shift-corrected distorted images (which we refer as coherent averaging of distorted images) can recover the missing Fourier phase information. The method is simple, fast, does not have any open parameters, is robust to noise, and we empirically find that the method does not fall into local minima. Our idea is grounded on the statistical properties of random waves where the coherent ensemble averaging of random PSFs has zero phase up to the diffraction limit, which is a key property that is also crucial for the bispectrum analysis.

## 2. Experimental Section

To describe the key principle of the method, the effects of aberrations and scattering were analyzed using the system effective PSF as follows,

$$I_{n}(x, y) = O(x, y) * P_{n}(x, y)$$
(3)

Here,  $P_n(x, y)$  is the *n*th distorted effective PSF that is randomly generated by turbulence or multiple scattering which can be also represented as,

$$P_{\rm n}(x, y) = P_{\rm ideal}(x, y) * R_{\rm n}(x, y)$$
(4)

 $P_{\text{ideal}}(x, \gamma)$  is the ideal diffraction-limited PSF (Airy disk) for an aberration-free optical system, and  $R_n(x, \gamma)$  is the *n*th random pattern that consists of sharply peaked waveforms generated by random disturbance (**Figure 1a**). To summarize, the distorted system's effective instantaneous PSF,  $P_n(x, \gamma)$ , is given by the convolution of the ideal PSF,  $P_{\text{ideal}}(x, \gamma)$ , with the instantaneous random position and weights,  $R_n(x, \gamma)$ , generated by random distortions. Equation (4) shows that the average speckle size of  $P_n(x, \gamma)$  still utilizes the full numerical aperture of the imaging system. In other words, the speckle PSFs have a common correlation length given by the average speckle size. However, when simply ensemble average  $P_n(x, \gamma)$ , this sharp feature is lost and a constant featureless blur is obtained (Figure 1b,c).

Let's now consider coherent averaging of  $P_n(x, y)$ . Coherent averaging of  $P_n(x, y)$  can be performed by shift correcting the randomly distorted  $P_n(x, y)$  such that  $P_n(x, y)$  always has its maximum value at the origin (it is emphasized that the term "coherent averaging" in this manuscript refers to shift corrected averaging of image intensities, not coherent adding and interference of optical fields). Then, the shift corrected  $P_{n,corr}(x, y)$  can be represented as,

$$P_{n,corr}(x, y) = P_{ideal}(x, y) * R_{n,corr}(x, y)$$
(5)

where  $P_{n,corr}(x, y)$  and  $R_{n,corr}(x, y)$  are the shift-corrected versions of  $P_n(x, y)$  and  $R_n(x, y)$ , respectively. Here,  $R_{n,corr}(x, y)$  can be further expressed as,

$$R_{n,corr}(x, y) = a'_{n}\delta(x, y) + R'_{n,corr}(x, y)$$
(6)

where  $a'_n \delta(x, y)$  are maximum peaks aligned at the origin with weighting factor  $a'_n$ , and  $R'_{n,corr}(x, y)$  are the remaining random terms of  $R_{n,corr}(x, y)$ .

By ensemble averaging  $P_{n,corr}(x, y)$ , *n* maximal peaks within the coherence length in the shape of  $P_{ideal}(x, y)*a'_n\delta(x, y)$  are coherently averaged, while other random peaks  $P_{ideal}(x, y)*R'_{n,corr}(x, y)$  at other areas beyond the correlation length are randomly averaged incoherently. In other words, the coherently ensemble averaged  $P_{n,corr}(x, y)$  consists of the sum of two parts; a sharp

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Figure 1. Principle of phase retrieval using coherent averaging. a) Randomly realized PSFs observed by an imaging system looking through random media can be expressed as a convolution of the diffraction-limited unit PSF and random patterns. b) In simple averaging, the averaged PSF is a constant function over space, which can be represented as convolution of the diffraction-limited unit PSF and a constant pattern. c) This result can be viewed in a different perspective by looking at the Fourier transform. The Fourier transform of the averaged PSF is a sharply peaked function, which results from the multiplication of a triangular function and a sharply peaked function. Therefore, the high spatial frequency information is lost. d) In coherent averaging, the averaged pattern is a sharply peaked function on top of a constant offset. This coherently averaged pattern is convolved with the diffraction-limited unit PSF, which is the coherently averaged PSF. e) In the Fourier domain, the coherently averaged OTF is the inverse Fourier transform of the product of the triangular function (diffraction limited OTF) and the sharply peaked function with constant offset. As the triangular function is multiplied by nonzero values over all frequencies, the resulting OTF has nonzero values up to the diffraction limit enabling Fourier phase retrieval. Red arrows correspond to the diffraction limit range in the spatial frequency domain.



**Figure 2.** Reconstruction flowchart.  $\tilde{I}_n(\vec{k})$ : Fourier transform of  $I_n(\vec{r})$ .  $I_0(\vec{r})$ : arbitrary image used as starting reference.  $I_{n,corr}(\vec{r})$ : shift-corrected *n*th image.

diffraction-limited focus and a diffuse background haze (Figure 1d,e) which can be expressed as,

 $\langle P_{n,corr}(x, y) \rangle = P_{ideal}(x, y) + constant background$  (7)

Because of the background haze, ensemble averaged shift-andadd images have a common diffuse and blurry look which has been a severe drawback limiting widespread use of the shift-andadd method despite its simplicity. However, it is shown that this limitation was due to the fact that the shift-and-add images were used as the final product in previous works. The hidden gem of this method, which has been somehow overlooked, lies in the Fourier spectra of the shift-and-add images. Since  $P_{ideal}(x, y)$ and the constant background are both real and symmetric, the Fourier transform of  $\langle P_{n,corr}(x, y) \rangle$  is real and therefore the Fourier phase of  $\langle P_{n,corr}(x, y) \rangle$  is zero. Combined with Equation (3), it can be seen that the Fourier phase of an object can be directly extracted by simply coherently averaging randomly distorted images. Although the shift-and-add PSF,  $< P_{n,corr}(x, y) >$ has a smaller Fourier amplitude for higher frequencies, it still does not fall to zero due to the sharp focus remaining above the background haze which allows to directly retrieve the Fourier phase up to the diffraction limit.

The entire workflow for the phase retrieval is illustrated in **Figure 2** as follows: (1) An arbitrary image is chosen as a reference image and cross correlation is performed between the reference image and all the other acquired images. (2) The relative shifts for all images are extracted and corrected for. (3) The shift-corrected





Figure 3. Experimental setup. Two separate setups were used to demonstrate our method in (1) imaging through dynamic turbulence generated by fire and (2) single shot speckle imaging through multiple scattering.

images are ensemble averaged from which the Fourier phase is extracted. (4) The retrieved Fourier phase is combined with the Fourier amplitude obtained using Labeyrie's method and inverse Fourier transformed to obtain the recovered image. (5) Repeat steps (1–4) using the recovered image as the reference image until convergence or target iteration number. It is noted that in contrast to conventional iterative phase retrieval algorithms where specific constraints are continuously applied in the spatial and Fourier domains during each iteration and the iterations are part of the actual phase retrieval process, the iteration in the algorithm is only required to aid in obtaining the correct shift corrections for coherent ensemble averaging. The phase retrieval in the method is achieved by a single Fourier transform of the coherent ensemble averaged image.

## 3. Results and Discussion

To demonstrate the validity of our phase retrieval method, we acquired randomly distorted images in two different experimental schemes (**Figure 3**). In the first geometry, we imaged fine structured objects through atmospheric turbulence where temporally random distortions were applied. To create such an imaging condition, we imaged a U.S. five-dollar bill illuminated by a green light emitting diode with central wavelength of 565 nm through severe random temperature/density variations generated by a gas burner. The distorted images were directly imaged through turbulence at a distance of  $\approx$ 1 m using a camera (LT545R, Lumenera) equipped with a 105 mm focal length camera lens (f/2.8, Nikkor) resulting in an effective magnification of 8.3. In the second geometry, spatially random distortions were generated using multiple scattering. A negative USAF target (Group number 1, Edmund Optics) was illuminated by a spatially incoherent narrowband light source which was generated by passing a coherent laser beam (532 nm, Shanghai Dream laser) through a dynamic speckle reducer (LSR-3005, Optotune). The scattered field then passed through a multiple scattering medium (ground glass, 220 Grit, Thorlabs) where the resulting distorted speckle image was recorded using a camera (IT545R, Lumenera) placed behind the scattering medium. The distances between the targetscattering medium and scattering medium-camera were 200 mm and 50 mm, respectively.

The image recovery is first initiated by recovering the Fourier power spectrum of the object using the distorted images via Labeyrie's autocorrelation method<sup>[4a,b]</sup> where the autocorrelation of each distorted image was Fourier transformed and ensemble averaged to obtain the Fourier power spectrum of the object. The second half of image recovery performs the Fourier phase retrieval. Our key finding in this work is that simple shift-andadding, or coherent averaging of the randomly distorted images actually holds information about the Fourier phase. To validate this property, we first imaged a sub-diffraction-limited pinhole as the test target. Short exposure images were acquired through severe dynamic turbulence using a gas burner to check its applicability in toughest conditions. Simply ensemble averaging the images resulted in a diffuse haze (Figure 4a). The effective PSF for the long exposure is in the order of  $\frac{\lambda}{r}$  and the spatial frequency cutoff is given by the inverse of the PSF. In stark contrast, coherent averaging results in a sharp focus on top of the



**Figure 4.** Comparison of simple averaging and coherent averaging. a) PSF obtained via simple averaging, b) PSF via coherent averaging, c) the Fourier phase of (a), d) the Fourier phase of (b). The white dotted lines are guides to the eye showing that simple averaging is limited to a cutoff frequency defined by the Fried parameter. In contrast, the Fourier phase of coherent averaged PSF is constant up to the diffraction limit. Scale bars: 30 camera pixels, corresponding to 275 µm at the object plane.

same diffuse haze (Figure 4b). The size of the sharp focus is indeed diffraction limited which proves that the spatial frequency cutoff is now extended up to the diffraction limit (the whited dotted line in Figure 4d is the diffraction-limited frequency cutoff as obtained when the turbulence was removed). As shown in Equations (5) and (6), the Fourier phase is constant for both ensemble averaged PSFs (Figure 4c,d), however, with different cutoff frequencies (see Movie S1 and Figure S3, Supporting Information for direct visualization of Fourier phase recovery upon coherent averaging and similar verification of resolution recovery for multiple scattering and Figure S4, Supporting Information for simulations demonstrating robust reconstruction under severe levels of turbulence).

Based on our observation, we next proceeded to fully recover diffraction-limited images of extended objects. **Figure 5** shows the final recovered images. All of the 50 images employed for the reconstruction were distorted by random turbulence (Figure 5a). Averaging the acquired images is equivalent to a single long exposure image that just averages out the high spatial frequency information (Figure 5b). By removing the relative shifts between the images and then averaging (conventional shift-and-add), we obtain an image that is slightly better than the simple sum (Figure 5c). Although conventional shift-and-add method results in a degraded image due to incorrect Fourier amplitude recovery, the Fourier phase is fully recovered up to the diffraction limit. For imaging through atmospheric turbulence, no iterations were required, and the Fourier transform of a single shift corrected coherently averaged image concluded the Fourier phase retrieval. Using the Fourier phase from the shift-and-add image combined with the Fourier amplitude recovered using Labeyrie's method, we obtain the fully recovered image (Figure 5d).

Since our method relies on cross correlation between randomly distorted images, a natural question we can ask is whether this method can be applied robustly to severely distorted images where the cross correlation between images is expected to be minimal. To answer this question, we performed single shot experiments through multiple scattering media. This experiment was designed to utilize the optical memory effect<sup>[12]</sup> in the socalled speckle correlation imaging geometry.  $^{\left[ 13\right] }$  In speckle correlation imaging, a lensless imaging configuration is utilized to transform angle tilts to spatial shifts based on far-field diffraction of light. Light emanating from a point source passes through a scattering medium and results in a random speckle intensity (the speckle PSF) distribution at the output observation plane. The optical memory effect states that if the impinging light field on a scattering medium is tilted, then the output speckle field is also tilted accordingly by the same angle within the memory effect range. Thus, light from a slightly shifted point source now





**Figure 5.** Imaging through turbulence. a) An example acquired image. b) Simple ensemble averaging of acquired images. c) Conventional shift-and-add ensemble averaging. d) Our proposed method. Insets in (c) and (d) show the recovered Fourier amplitude compared with the diffraction limit outlined in yellow dotted lines. Scale bars: 60 camera pixels, corresponding to 498 μm at the object plane.

generates an output speckle field emanating from the scattering medium that is the same except for an additional angle tilt. The final speckle distribution observed at the far-field beyond the scattering medium is then simply translationally shifted proportionally to incident beam tilt angle due to far-field diffraction. Therefore, we can see that an extended object, or a sum of incoherent point sources, behind the scattering medium generates multiple plane waves with their respective angle tilts that impinge onto the scattering medium. The resulting output total speckle intensity at the observation plane now simply becomes an incoherent sum of shifted speckles. In other words, the observed total speckle intensity distribution is simply the convolution of the object with the speckle intensity that is observed for a single point source (the speckle PSF), equivalent to Equation (3).

As we anticipated, the acquired image in imaging through scattering media was much more severely distorted compared to atmospheric turbulence. We first checked that ergodicity is valid in our multiple scattering medium. **Figure 6**a shows the original object and its autocorrelation. After passing through multiple scattering media, a fully developed speckle is imaged on the camera (Figure 6b). By taking the autocorrelation of the speckle, the original object autocorrelation can be recovered due the optical memory effect (Figure 6c).<sup>[12–14]</sup> Furthermore, similar with holographic data storage, distinct subregions of the speckle pattern also hold information about the entire object, although with independent speckle footprints.<sup>[13c]</sup>

By dividing the full speckle pattern into multiple subregions  $(60 \times 60 \text{ pixels each})$ , we performed autocorrelation of each subregion followed by ensemble averaging. As shown in Figure 6d, the result is identical with that obtained using the full speckle as well as the original object Fourier spectra verifying that the ergodic nature of multiple scattering is valid in our experimental setup.

Based on ergodicity of multiple scattering, a single image was sufficient to recover the object. Instead of using randomness realized as a function of time as in atmospheric turbulence, the randomness of multiple scattering in space was utilized by using subimages as independent realizations (**Figure 7**a). The acquired single image was empirically divided into partially overlapping subimages with sizes of  $60 \times 60$  pixels each to obtain 133 533 subimages for coherent averaging. These subimages correspond to individual randomly distorted image frames in imag-



**Figure 6.** Recovery of object power spectra. a) The ground truth autocorrelation of the object. The ground truth object is shown in the inset. b) Imaged speckle pattern. c) Autocorrelation of the speckle pattern (b). d) The speckle pattern (b) was divided into subimages with a size of  $60 \times 60$  pixels each. The ensemble average of the autocorrelation of each subimages is identical with (a) and (c). Scale bars: 20 camera pixels, corresponding to  $670 \,\mu\text{m}$  at the object plane.

ing through turbulence. When simply averaging all the subimages, information about the object was just averaged out resulting in an information-less constant background (Figure 7b). In comparison with imaging through turbulence, we found that shift correcting the speckle like random images through scattering media was more challenging as expected. The shift correction was therefore iteratively optimized by simply using the shift corrected ensemble averaged image obtained from the previous round of iteration as the reference image for the next round of iterative shift corrected coherent averaging. After employing 5 iterations of coherent averaging of the distorted subimages (shift-and-add), useful information was retrieved similarly to imaging through turbulence (Figure 7c). By using the Fourier phase from the iterative coherently averaged image in combination with the Fourier magnitude extracted from the ensemble averaging of distorted subimage autocorrelations, we successfully recovered the diffraction-limited resolution image (Figure 7d) even through multiple scattering media (Movie S2, Supporting Information).

# 4. Conclusion

We have demonstrated a new method for phase retrieval using randomly distorted images simply by exploiting the Fourier phase information that is recovered by using the shift-and-add method which has previously never been realized. To date, the biggest difficulty in realizing computational imaging through random media has been related to the accurate Fourier phase retrieval. In contrast to the well-established formalism for obtaining the Fourier amplitude which generally follows the initial idea proposed by Labeyrie, research related to developing effective iterative phase retrieval algorithms are still very actively ongoing. This reflects the difficulty of the problem and the fact that current existing techniques have various limitations that must be overcome. While previous approaches in iterative phase retrieval require assumptions about the object beforehand such as nonnegativity and constraints on the extension area of the object, our method does not require pre-assumed constraints or initial guesses and is fast with almost no excess computational load.



**Figure 7.** Imaging through multiple scattering media. a) An example acquired subimage. The total scattered image can be seen in Figure 2(b). b) Simple averaging of subimages. c) Iterative coherent averaging of the acquired subimages. d) Our proposed method. Insets in c and d show the recovered Fourier amplitude. Scale bars: 20 camera pixels, corresponding to 670  $\mu$ m at the object plane.

The process only consists of Fourier transforms and addition of the acquired images. It is also straightforward to parallelize the method for large images and makes this a powerful method for computational image reconstruction of extended field of views that are larger than the isoplanatic patch.<sup>[15]</sup> For example, a large object of interest where different areas have undergone different distortions can be independently analyzed and then simply recombined to obtain images that are larger than a single isoplanatic patch size. Using this approach, we demonstrate large field of view (910 × 730 pixels) real-time image reconstruction through severe turbulence generated by fire at  $\approx$ 45 Hz refresh rate (Movie S3, Supporting Information). We have found the method to be robust in various degrees of randomness ranging from atmospheric turbulence to multiple scattering as well as related open datasets (see Figure S5, Supporting Information). The method has no open parameters that need to be optimized and a single general-purpose algorithm (without any modifications) was found to be effective for various data sets. In comparison with previous methods, the memory requirement and reconstruction time for recovering a  $100 \times 100$  pixel image are merely 80 kB and 0.011 s using our method while the bispectrum analysis requires 1.6 GB of memory and 5940 s for reconstruction of the same image (see Figure S6 and Table S1, Supporting Information for performance comparison with the bispectrum analysis and Figure S7 and Movie S4, Supporting Information showing phase retrieval convergence). We believe that the simplicity, robustness, and accuracy of our method has potential to open new avenues for phase retrieval in randomly distorted data. Considering that time-varying distortion is one of the major limitations in in vivo deep tissue imaging, our method can also provide a breakthrough by exploiting the random fluctuations for diffraction-limited image reconstruction through live tissues.

# **Supporting Information**

Supporting Information is available from the Wiley Online Library or from the author.

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### **Conflict of Interest**

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The authors declare no conflict of interest.

#### **Data Availability Statement**

The data that support the findings of this study are available in the supplementary material of this article.

#### **Keywords**

multiple scattering, phase retrieval, speckle interferometry, turbulence

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