

Low-complexity compressive sensing with downsampling

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Abstract: Compressive sensing (CS) with sparse random matrix for the random sensing basis reduces source coding complexity of sensing devices. We propose a downsampling scheme to this framework in order to further reduce the complexity and improve coding efficiency simultaneously. As a result, our scheme can deliver significant gains to a wide variety of resource-constrained sensors. Experimental results show that the computational complexity decreases by 99.95% compared to other CS framework with dense random measurements. Furthermore, bit-rate can be saved up to 46.29%, by which less bandwidth is consumed.

Keywords: compressive sensing, downsampling, sparse random matrix, low-complexity, sparse signal recovery

Classification: Electron devices, circuits, and systems

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1 Introduction

Our daily lives have been permeated by various sensing devices ranging from mobile phones to biosensors. Most natural or man-made signals have correlations that can be translated into a certain structure when captured by sensors. Conventional source coding schemes leverage this structure via transform coding, which represents the signal with only a few principal components. These principal components along with side information are entropy-coded using variable-length codes such as arithmetic coding. However, this near-optimal coding process is not applicable to many resource-constrained devices due to its complexity.

Compressive sensing (CS) shifts the complexity burden of conventional source coding from encoder to decoder with reasonable expense in coding efficiency [1, 2]. Thus CS methods can be applied to various types of resource-limited sensors such as wearable devices [3]. In CS, a signal is projected onto random sensing basis, which is essentially the computation of inner products, that is, multiplication and summation operations.

The sparse random matrix significantly reduces CS encoding complexity while assuring the same performance as dense random matrices that are prevalent in CS [4]. This sparse random matrix is a binary and sparse matrix used for random sensing basis of CS.

In this paper, we aim to reduce the complexity of these operations further by incorporating downsampling into CS with the sparse random matrix. We show the downsampling not only reduces the computational burden of CS-implemented sensor devices dramatically, it also improves the coding efficiency of CS when combined with linear interpolation.

Experimental results demonstrate that our downsampling approach outperforms existing CS framework, which can be translated into more resource savings if we were to retain the same data quality. As a result, resource-limited sensing devices can benefit from our low-complexity CS approach without compromising coding performance.

2 Low-complexity CS with downsampling

Consider a time-domain signal $\mathbf{x} \in \mathbb{R}^N$ captured by a sensor that can be compactly represented in some orthogonal basis Ψ with only a few large coefficients (principal components) and many small coefficients close to zero, which is a typical scenario in real-world sensing. We can project \mathbf{x} onto random sensing basis $\Phi \in \mathbb{R}^{M \times N}$ as follows [5]:

$$\mathbf{y} = \Phi \mathbf{x} = \Phi \Psi \mathbf{s}, \quad (1)$$

where the transformed signal \mathbf{s} is K -sparse (K large coefficients).

In Eq. (1), Φ is generally constructed by sampling independent identically distributed (i.i.d.) entries from the Gaussian or other sub-Gaussian distributions that have more uniform and shorter tail than Gaussian (e.g., Rademacher distribution) [6]. (The moment-generating function of a sub-Gaussian distribution is bounded by that of a Gaussian.) Consequently, Φ

is dense with virtually every entry set to non-zero real numbers. This leads to $O(MN)$ multiplication and summation operations; however, this can be costly to resource-limited sensors without specific CS-supporting architectures [6].

The sparse random matrix turns out to be a solution to this complexity issue. The random sensing matrix Φ now has d ones for each column; and all other entries are zeros. (Each column has roughly the same number of ones: slight unbalance in the number does not affect overall results [4].) It was shown that this matrix construction could be deemed an adjacency matrix of an *unbalanced expander graph*, which at the same time satisfies RIP-1 (*restricted isometry property*) [4]:

$$(1 - \delta)\|\mathbf{s}\|_1 \leq \|\Phi\Psi\mathbf{s}\|_1 \leq (1 + \delta)\|\mathbf{s}\|_1, \quad (2)$$

where $\delta > 0$ should not be close to one [6]. Note that Φ constructed using the Gaussian or sub-Gaussian distributions satisfies RIP-2, i.e., the ℓ_2 norm instead of the ℓ_1 norm in Eq. (2). It was also shown that the sparse random matrix satisfying RIP-1 was essentially as good as dense matrix satisfying RIP-2 [4]. Furthermore, a decoder with the RIP-1 matrix can recover the original signal using linear programming as in the case of RIP-2 matrix, which is given by

$$\min\|\tilde{\mathbf{s}}\|_1 \quad \text{subject to} \quad \Phi\Psi\tilde{\mathbf{s}} = \mathbf{y}. \quad (3)$$

The solution \mathbf{s}^* to Eq. (3) obeys

$$\|\mathbf{s}^* - \mathbf{s}\|_1 \leq C \cdot \|\mathbf{s} - \mathbf{s}_K\|_1 \quad (4)$$

for some constant C , where \mathbf{s}_K is the vector \mathbf{s} with all but the largest K components set to 0: the quality of recovered signal is as good as that with the K most significant pieces of information [4, 6]. We get progressively better results as we compute more measurements M [7].

Because of the selective nature of the sparse random matrix, computational complexity is reduced to $O(dN)$, where $d = O(\log(N/K))$ [4, 8]. This is a considerable saving compared to the general case of $O(MN)$, where $M = O(K \log(N/K))$. In fact, we found that d could be decreased as small as 2 without noticeable loss in coding efficiency from our experiments where two different signal types were used. (If $d = 1$, a subset of K columns taken from Φ can be linearly dependent when $M < N$ since there can be at most $\binom{M}{1}$ unique columns.)

We now introduce the *downsampling* scheme, which is the main contribution of this paper, so as to further reduce the computational complexity and increase the coding performance at the same time. Fig. 1 presents our low-complexity CS architecture. The downsampling process takes every L th sample and the upsampling process inserts $L-1$ zeros between samples, where L is a downsampling factor. Note that the sparse random matrix generation can be synchronized between encoder and decoder using pseudorandom number generator, which is a common practice in CS literatures [7].

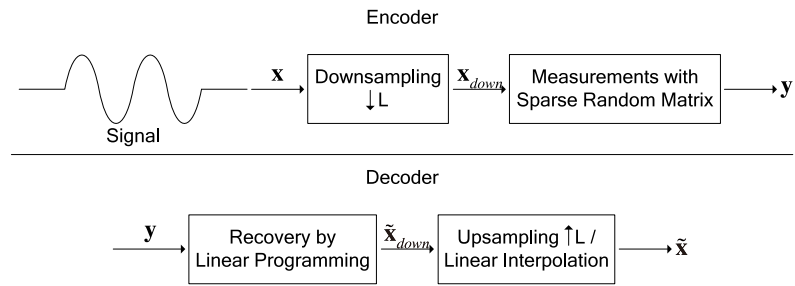


Fig. 1. Low-complexity CS architecture incorporating downsampling.

Another important thing is that our downsampling at the encoder does not involve prior low-pass filtering, which inevitably incurs *aliasing* of the signal. However, we empirically found that using low-pass filters (LPFs) generally introduced much distortion when up-sampled and linear-interpolated. This can be attributed to the fact that real LPFs do not compare with an ideal LPF in terms of sharp cutoff between passband and stopband.

The downsampling in Fig. 1, combined with upsampling and linear interpolation, yields better coding performance than general CS framework. The rationale behind the better coding performance with downsampling is illustrated in Fig. 2, where original sensor data and two approximations using CS and CS with downsampling are drawn together. We can identify that down-sampled approximation is smoother than general CS approximation, resulting in less distortion. In other words, CS recovery tries to approximate the original signal while incurring distortion bounded by Eq. (4), which can be mitigated by less sample points recovery and smoothing out fluctuations using linear interpolation.

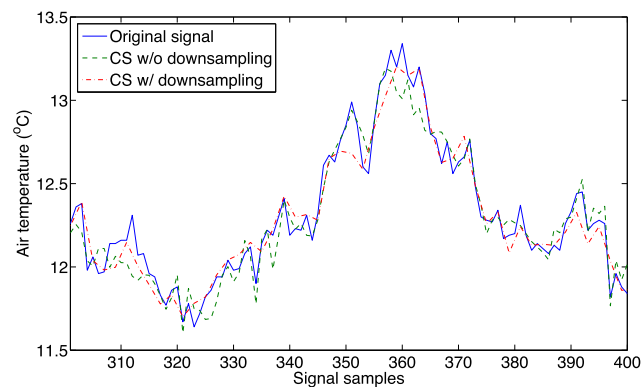


Fig. 2. Air temperature data [9] and its approximations using CS with and without downsampling (Daubechies-8 wavelet for Ψ).

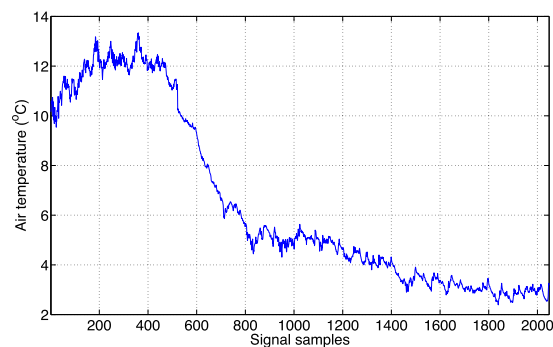
The downsampling scheme further reduces the encoding complexity to $O(dN/L)$. We classify overall encoder complexities in Table I.

Table I. Overview of encoder complexities.

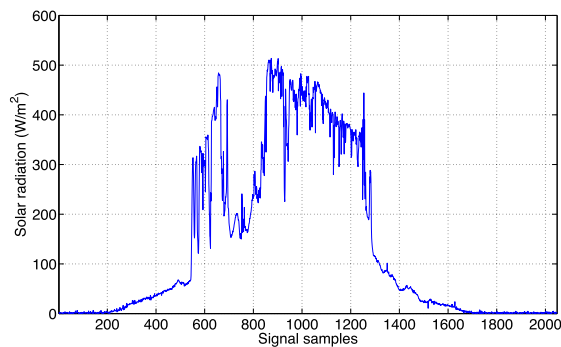
General CS	Sparse Random Matrix	Our Scheme
$O(NK \log(N/K))$	$O(N \log(N/K))$	$O((N/L) \log(N/K))$

3 Experimental results

Two different signal types from environmental sensor data set shown in Fig. 3 were selected for our experiments [9]. In Fig. 4, we show averaged results of our downsampling scheme and the baseline scheme without downsampling. We here consider sum of squared error (SSE) distortion; parameters of the sparse random matrix are $M = 1024$, $N = 2048$, and $d = 2$. It should be noted that in Fig. 4, the performance of baseline scheme is equivalent to general CS framework that uses dense Gaussian matrix for random sensing basis.



(a)

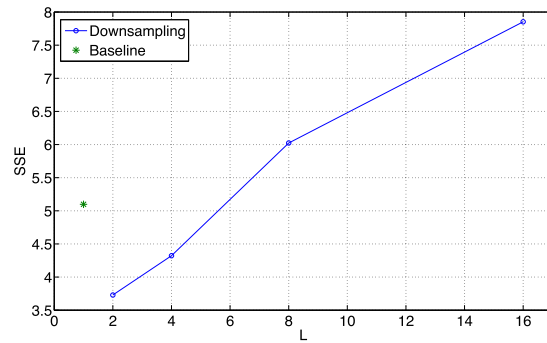


(b)

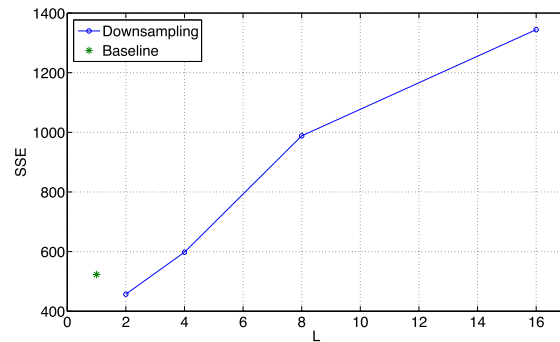
Fig. 3. Environmental sensor data of (a) static and (b) dynamic types.

In Fig. 4, the extra benefit of our scheme appears at $L = 2$ (and 4); however SSE increases after this point, which means too few sample points and interpolation between them oversimplify approximations. Obviously, if reducing the computational burden is the utmost importance, a sensing device can increase the downsampling factor while sacrificing data quality.

Meanwhile, we obtain these results using dN/L computations as compared to MN computations in general CS framework, which especially is



(a)



(b)

Fig. 4. SSE comparison with several downsampling factors for (a) air temperature and (b) solar radiation data.

99.95% of reduction at $L = 4$. Furthermore, we can leverage this benefit to reduce the length of vector \mathbf{y} , which corresponds to rate and bandwidth usage of sensors. Therefore, we can find the minimum number of measurements that allows the same SSE as the baseline measurements. The resulting rate savings were 46.29% for “air temperature” data and 32.62% for “solar radiation” data.

4 Conclusion

This paper proposes a low-complexity CS that is suitable for resource-constrained sensing devices. We can dramatically reduce typical encoding complexity of CS, employing both sparse random matrix and downsampling scheme. Moreover, we have shown that extra coding efficiency from downsampling can be transformed into extra rate savings. We plan to extend the downsampling approach experimented temporally within individual device to spatially distributed sensing domain.

Acknowledgments

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