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Observation of trapped-modes excited in double-layered symmetric electric ring resonators

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We report on experimental observations of trapped-mode resonances in double-layered symmetric electric ring resonators separated by dielectric inserts. The resulting metamaterial introduces trapped-mode resonances that were thought to be produced only by asymmetric metamaterial unit cells. Experimental verification of the newly observed trapped modes, along with the analysis of the stacked metamaterial geometry reported in this paper, opens an alternative way of forming sharp resonances in a symmetric metamaterial structure extended in all three dimensions. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4721993]

I. INTRODUCTION

Tremendous progress has been made in so-called “metamaterials” that manipulate the characteristics of a broad spectrum of electromagnetic waves.1–6 Potential applications of metamaterials such as absorbers and filters have led to sophisticated studies on various types of metamaterials over a broad range of frequencies, particularly terahertz and infrared bandwidths.7–10 The importance of understanding the frequency response of external EM waves has been recognized, and this has accelerated fundamental research on frequency splitting, broadband characteristics, blue-shifts of resonance frequencies, and polarizations of metamaterials.11–15

For practical purposes, three-dimensional structures of metamaterials have been pursued recently.8 Layering metamaterials to create three-dimensional structures is thought to be a natural step in understanding the physics of multilayered metamaterials. In this context, researchers have reported frequency splitting resulting from multilayering.13,16

Experimental observations of so-called “trapped modes” were first reported by Fedotov et al.17 pursued after a theoretical analysis by Prosvirnin et al.18 Fedotov et al. observed sharp resonances using a structure involving asymmetrically split rings (ASR) by analyzing symmetric current modes. The idea was further refined by applying split ring resonators (SRRs) to observe electromagnetically induced transparency (EIT) in the X-band by exciting trapped modes.19–26

To date, trapped modes for which the current distribution is same in magnitude with opposite direction between the unit cells have occurred only in asymmetric geometries of “in-plane” metamaterial unit cells. Here we report on multiple trapped modes in a double-layered metamaterial configuration consisting of electric ring resonator (ERR) unit cells separated by dielectric slab inserts. With such components, the current distribution on the top and bottom of the metamaterial cells is same in magnitude with opposite direction between the unit cells with respect to the center of the two cells in the propagation direction.

II. RESULTS AND DISCUSSIONS

We fabricated ERR patterns on a FR4 substrate. Each layer of ERR metamaterial has dimensions 20 cm × 20 cm and consists of a total 782 ERR unit cells (46 × 17 cells). The detailed dimensions of an ERR are given in Fig. 1(a). An Agilent PNA-X N5242A was connected to dual-polarized quad-ridge X-band horn antennas (AS-48461, EDO, Inc.) that provide dual polarized signals for our experiments. For the study of EM response of different polarization effects, we used the convention that the electric fields propagate along the z-axis direction with the y-axis polarization (denoted ypol) and x-axis polarization (denoted xpol), as shown in Fig. 1(b). In between the ERR metamaterial plates, up to four 0.6 mm-thick bare FR4 dielectric slabs were inserted and stacked so that the plate separation could be varied. Fig. 1(b) depicts the experimental scheme for the stacked FR4 inserts between the ERR single layers. The fabricated stacked ERR layers were situated in the middle of the antennas for the transmission measurement as shown in Fig. 1(c). For reflection measurement, the T/R antennas were put next to each other pointing to the ERR sample with slight angle so that the electromagnetic waves off the antennas were focused at the location where the ERR sample was located as depicted in Fig. 1(d). An aluminum plate was used as a reference for representing perfect reflection.

Extensive simulations were performed using the commercial code HFSS v12 to compare with experimental results.
A Floquet port setting was used for the repetitive ERR structure analysis with an assumption of dielectric constant, $\varepsilon = 4.4 + i0.02$, for the FR4 substrate at X-band frequency in the simulation.

Fig. 2 shows the results of simulations for the transmission and reflection spectra of stacked FR4 inserts. Figs. 2(a)–2(d) give transmission data and Figs. 2(e)–2(h) give reflection data corresponding to 1, 2, 3, and 4 bare inserts between the ERR layers. Fig. 3 shows the experimental spectral results of transmission and reflection of the manufactured ERR cell slabs. Comparing Figs. 2 and 3, the experimental transmission and reflection data are in agreement with simulation data for both ypol and xpol. In the experimental transmission spectra, frequency splitting is observed for ypol for all four separations. In contrast, two peaks in the xpol transmission spectra appear at around 10 GHz and 14 GHz as the separation increases. These match well with simulation results although there is slight frequency mismatch in peaks between experiment and simulation. Experimental reflection spectra are also well described in simulations in terms of location and number of peaks for both ypol and xpol. From the transmission and reflection data of simulation and experiment, the absorption spectra were obtained from $A = 1 - T - R$ as shown in Fig. 4. High absorption was observed, which is originated from energy dissipation in the substrate that holds the electromagnetic energy as trapped modes. Good agreement between the simulation and the experiment was found in absorption spectra as shown in Fig. 4.

Fig. 5 shows the simulated surface current density $J_{surf}$ induced on the metal surfaces of the ERR structures. Each $J_{surf}$-plot corresponds to a frequency marked in the panels of Figs. 2 and 3. The transmission and reflection spectra are strongly affected by the polarization of the incident electromagnetic wave. For incident ypol waves, resonance properties occur at lower frequency ranges (between 9 GHz and 15 GHz) with the accompanying frequency splitting. However, for these resonances in ypol, not only transmission but also reflection resonances occur at dips in the spectra; thus, the interpretation is that the incoming wave is strongly absorbed in the ERR samples due to a strong coupling between both.

New phenomena in the frequency response occur with xpol. For 1 bare insert, a transmission resonance peak appears at the marked frequency 1Ax at around 11 GHz (see Fig. 2(a)). The corresponding reflection data show a resonance dip at this frequency; the top-left panel in Fig. 5 gives the $J_{surf}$-plot. A strong current flow is seen as the incoming xpol wave excites the electric field along the x-axis, which corresponds to the alignment of the top and bottom arms of the first ERR layer. For the second ERR layer, the current density vector changes direction, thus forming a symmetric pattern with respect to the mid-point between the two ERR layers. Mathematically, we have $J_1(-x, -z) = J_2(x, z)$ with opposite direction, where the subscripts 1 and 2 indicate the top and bottom arms of the ERR structures, respectively. This is interpreted as a “trapped mode,” or “closed mode” for which the wave is weakly coupled spatially.

![FIG. 1. A double-layer ERR structure on a FR4 substrate: (a) A unit cell with ERR located top and bottom and two FR4 plates in between. The dimensions of the ERR unit cell are $a_1 = 3.9\text{ mm}$, $a_2 = 3.9\text{ mm}$, $\delta = 12\text{ mm}$, $w = 4.2\text{ mm}$, $t = 0.6\text{ mm}$, and $g = 0.606\text{ mm}$. The thickness of the copper ERR pattern is 17 $\mu\text{m}$. In (a), it is shown that two FR4 plates are inserted in between two ERR layers, which are facing in the same direction. (b) Schematic of the unit cell used in the FR4-stacked experiment, in which from one to four FR4 plates were tightly stacked at various thickness by dielectric inserts. The two ERR cells are facing the same direction. The $z = 0$ plane is situated mid-way between two ERR cells, each located at distance $z_0$ from the center point. (c) Transmittance measurement setup. (d) Reflectance measurement setup.](image)

![FIG. 2. Simulated transmission (a)–(d) and reflection (e)–(h) spectra of a double-layer ERR structure with various separation distances made with bare FR4 dielectric inserts. From left to right the number of bare inserts increases from 1 to 4, as seen labeled above each column of panels. The solid blue and dotted red lines represent ypol and xpol, respectively; green vertical lines indicate trapped modes.](image)
Trapped modes were recently reported in metamaterials using asymmetric structures by slightly breaking the structural symmetry. Symmetric current distributions with opposite direction were found in such structures with narrow resonance features in the transmission and reflection spectra.\textsuperscript{17,20,24} In these earlier trapped-mode studies, a symmetric current distribution with opposite direction formed in the asymmetric SRR of the unit cell. Here, we have found trapped modes in symmetric metamaterial structures for which current distributions are symmetric with opposite direction with respect to the propagation direction.

Increasing the separation between ERR layers to two bare inserts, one can also find similar trapped modes. Labels 2Ax and 2Cx mark transmission resonance peaks where the corresponding reflection spectra show dips (see Figs. 2(f) and 3(f)). The associated $J_{\text{surf}}$-plots in Fig. 5 satisfy the symmetric current mode condition, $J_1(-x, -z) = J_2(x, z)$ with opposite direction. However, midway between the frequencies for 2Ax and 2Cx appears another resonance peak, labeled 2Bx, featuring a transmission minima and a reflection maxima. The $J_{\text{surf}}$-plot for 2Bx exhibits small $J_{\text{surf}}$ amplitudes with no indication of a symmetric current distribution with opposite direction, i.e., no trapped mode.

Interestingly, a ypol trapped mode is observed at much higher frequency in harmonic resonance labeled 2Ay in Fig. 2. From the corresponding $J_{\text{surf}}$-plot in Fig. 5, the current distribution is seen not to be a fundamental resonance excitation, but a strong electric field occurs across the gaps of the ERR aligned with the ERR plane. The surface current density direction in second ERR layer has changed direction, yielding the same symmetric pattern as observed in trapped modes. Similar to two bare inserts, the same features in xpol and ypol are observed with three bare inserts. The resonances labeled 3Ax and 3Cx in xpol satisfy trapped mode conditions. Also, the resonance 3Ay in ypol is found to be a trapped mode with a very sharp peak in the reflection spectra, but resonance 3By is not. For 4 bare insert, the current density distributions for 4Ax and 4Cx show slight asymmetry, the greater asymmetry in 4Cx arising because of weaker reflection spectra compared with that for 4Ax. Nevertheless, one finds that there is still a strong trapped mode in ypol corresponding to resonance 4Ay.

As reported in Refs.\textsuperscript{22} and \textsuperscript{23}, an important characteristic of a trapped mode is a high Q factor. The multi-layered symmetric metamaterials show Q factors at resonances 1Ax (10), 2Ax (12), 2Cx (7), 2Ay (70), 3Ax (5), 3Cx (4), 3Ay (42), 4Ax (8), 4Cx (<5), and 4Ay (71), all with symmetric current distributions with opposite direction (parentheses indicate Q values) obtained from up to $-3$ dB bandwidth in reflection peak. In contrast to asymmetric planar metamaterials, as discussed in literatures,\textsuperscript{21–24} the trapped modes...
observed in symmetric but multi-layered metamaterials tend to have low Q factors due to the stacking of dielectric structures that result in losses. In simulations, we find that Q factors (not presented in this letter) increase more than two-fold (for resonance 2Ax, Q increases 5 fold) if dielectric inserts are not placed between ERR layers.

In short, Table I collates the trapped and non-trapped modes observed at resonances based on transmission and reflection resonance features. The symmetry pattern of the current distributions, as seen in the plots in Fig. 5, identifies the central characteristic of trapped and non-trapped modes.

The novelty of the observed trapped modes lies in symmetrical structure composed of the ERR cells whereas typical trapped modes are found in asymmetric structures. As shown in Fig. 5, the surface current density direction on each ERR layer indicated by blue arrows is symmetric with respect to the middle point ((x,y,z) = (0,0,0), the coordinate system was defined in Fig. 1) of two ERR structures, and the surface current density, \( J_{surf} \) is the same in magnitude but opposite in its direction, which results in weak scattered electromagnetic field. As a result, the net energy leakage to the free space from the ERR cells is small; in this case, the excited modes are trapped in the ERR slabs.

III. CONCLUSIONS

In summary, we investigated new phenomena of trapped modes excited in a wave propagation direction. Compared with “in-plane” trapped modes studied recently by researchers, the quality factor of these newly observed coupled trapped modes is smaller due to losses in the dielectric media. However, owing to the nature of the frequency splitting in multi-layered metamaterials, we were able to observe multiple trapped modes exhibited in a double-layer metamaterial, in which top and bottom ERR layers are separated at fixed distances. One can “tune” the thickness between two metamaterial structures, as well as its material properties, so that resonances of trapped modes become sharper. This study represents the first observation of multi-plane trapped modes for a symmetric ERR geometry, possible applications of which rely on further investigations.

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